MULTI-TANK LEVEL CONTROL USING INTERNAL MODEL CONTROL AND INTERNAL MODEL CASCADE CONTROL

Thesis submitted in partial fulfillment of the requirements for the degree of

Master of Technology

In

Electronics &Instrumentation Engineering

By

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Rourkela, Odisha -796008

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NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA

CERTIFICATE

This is to certify that the thesis report titled **"STUDY OF INTERNAL MODEL CONTROL AND INTERNAL MODEL CASCADE CONTROL"** Submitted by **Mr. Aishvarya Pratap Singh (**Roll No: 213EC3229) in partial fulfillment of the requirements for the award of Master of Technology in the Electronics and Communication Engineering with specialization in **"Electronics and Instrumentation Engineering"** during Session 2013-2015 at National Institute of Technology, Rourkela and is an authentic work carried out by him under my supervision and guidance.

……………………………...

Prof. Tarun Kumar Dan

Place: Dept. of Electronics and Comm. Engineering

Date: National Institute of Technology

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Aishvarya Pratap Singh

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CONTENTS

ABSTRACT

This project deals with the study, design and application of IMC (Internal Model Control) and IMCC (Internal Model Cascade Control) strategies for an unconstrained single input and single output system. Both strategies have been studied by applying them on a Tank Level Control System under varying system as well as input parameters. The application of both strategies on the model of the system is done and step responses of the modeled system under different operating conditions are analyzed to deduce various conclusions. Based on various performance parameters a tuning rule for best tuning parameter to give optimal performance has been designed. It also consist the comparison of various control strategies like PID and PID cascade with above two strategies with different tank configurations. An improvement is also added to IMC for improvement in disturbance rejection.

IMC consists of single tuning parameter which is filter coefficient of the main loop controller whereas IMCC consists two tuning parameters, one of primary loop and other for the secondary loop.

An important thing is that for designing of IMC and IMCC controllers modeling of the experimental setup has been done. The MATLAB and SIMULINK software has been used for designing of the IMC and IMCC controllers, which were designed considering deduced process model as original process. For IMCC, the secondary process is taken to be control valve.

At last an Empirical formula for IMC has been derived which gives the value of best tuning parameter for a given process, based on given performance indices like Rise Time, Settling Time, Peak Time and Peak Overshoot.

LIST OF FIGURES

LIST OF ACRONYMS

IMC	Internal Model Control
IMCC	Internal Model Cascade Control
PID	Proportional Integral Derivative
DAQ	Data Acquisition

CHAPTER-1

INTRODUCTION

1.1 LITERATURE SURVEY

1.2 OBJECTIVE

1.3 THESIS OUTLINE

1.1 Literature Survey

Coleman Brosilow, Babu Joseph have proposed a methods of model based control for IMC cascade control system [9]. They designed 1DF and 2DF controller to increase the performance of the IMC cascade control system.

Aravind, P., M. Valluvan, and S. Ranganathan have proposed the modeling procedure of a Non-Linear tank. They proposed strategy to linearize the whole non-linear scale by breaking it into small approximately linear scales [2].

Ming T. Tham has proposed the designing procedure of internal modal control method [8]. He defined the IMC strategy, basic principal, IMC based PID controller design approach.

Mishra, Rakesh Kumar, and Tarun Kumar Dan have proposed design procedure of Internal Model Control for distillation column [4]. They also proposed strategy to improve the disturbance rejection.

B. Wayne Bequette have proposed the Process control modeling, design and simulation for the cascade control system [10]. He defined the tuning of primary and secondary controller to cascade control system.

1.2 Objective

The objective of this thesis is to design an IMC and IMCC structure and Compare the output response of the PID, PID Cascade, IMC and IMCC to a step set-point change and step disturbance. To give a rule for the tuning of IMCC to get the optimal set point tracking as well as disturbance rejection..

To minimize the effect of disturbance on the primary process of the cascade control system through the operation of a secondary or inner control loop about a secondary process for desired operation and to design an empirical formula for giving best tuning parameter with given performance indices.

1.3 Thesis Outline

This thesis involves 5 chapters. After the introduction, the remaining portion of the thesis is organized as follows:

Chapter 2 IMC System

In this chapter IMC construction and properties has been discussed. The design method of IMC as well as tuning is also discussed here. It also contains the introduction to experimental setup and modeling of that system. Those models are then used to design the controller and the simulation results for different tuning parameters are discussed.

Chapter 3 IMC Based Cascade Control System

In this chapter the basic configuration of the cascade control system has been discussed. This cascade structure is then designed for IMC to get Internal Model Cascade Control. IMCC is presented as a means to compensate dominant secondary disturbances. It also contains Simulation results and discussions for the designed IMCC.

Chapter 4 IMC Based PID

In this chapter PID design procedure based on IMC is presented. Empirical formula for this controller has been then evaluated to give the value of desired tuning parameter for given values of performance indices.

Chapter 5 Conclusion and Future Work

Here the conclusions based on observations in different chapters have been discussed.

CHAPTER-2

IMC SYSTEM

2.1 INTRODUCTION

In this chapter we first develop an open loop control design procedure that is then used for the design of Internal Model Control. IMC is a form of model based procedure where the process model is embedded in the controller. By the knowledge of process as process model, improved performance can be obtained. The designing method of the controller has been performed, where the output of an instinctively stable process to perform a set-point variation and reduce the impact of disturbances that are added directly into the output of the process. IMC has many advantages compared to conventional feedback control structure. One of the main advantage is that they are much easier to tune. Here, we have a calculated model of the process, which permit us to predict the response of the process output to the disturbances and to control effort. Now we consider a process model that is connected in parallel to original process and receives the same manipulated variable as the actual process.

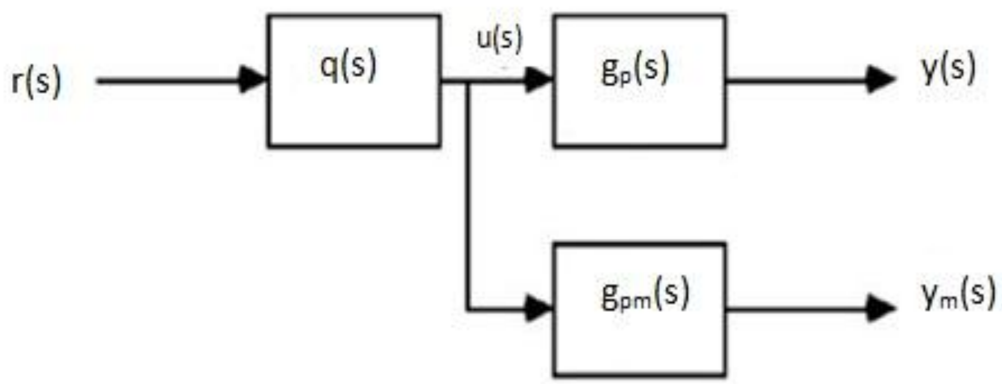

Figure 2.1 The Open Loop IMC System.

The various parameters used in the IMC system shown above are as follows:-

$r(s)$ = Set-Point

$q(s)$ = IMC controller

$g_p(s)$ = Process

$g_{pm}(s)$ = Process Model

$u(s)$ = Manipulated Variable

$y(s)$ = Process Output

$y_m(s)$ = Model Output

We can now subtract the difference between process output and model output to give error signal which refers to model mismatch. Now that error output is fed back to the input as a negative feedback. The final IMC structure is shown in the below Figure (2.2). IMC allows us to use the controller without worrying about the stability of the control system if the process is stable.

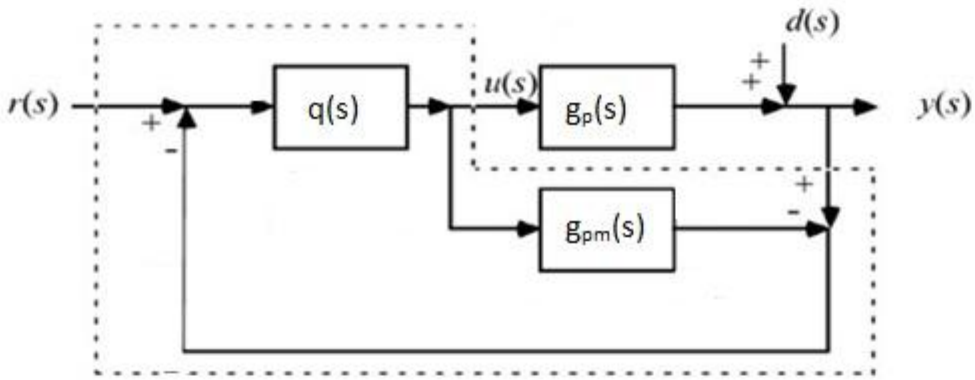

Figure 2.2 The IMC System

The dotted line indicates the portion where all the calculations performed by the model-based controller. Reordering the above diagram the alternate IMC configuration system is shown in the Figure (2.3).

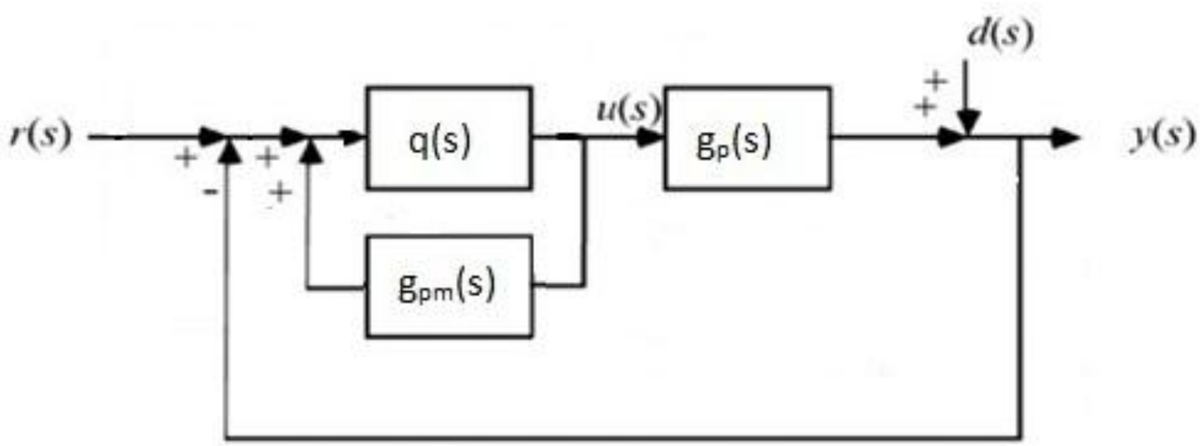

Figure 2.3 Alternate IMC Configuration Systems.

2.2 PROPERTIES OF INTERNAL MODEL CONTROL

2.2.1 IMC Background

Model based control is most efficiently used for the process with finite propagation delays. A basic example of this type of process is level control of a tank by controlling inlet and outlet flow. It is getting very popular now a days due to its effectiveness and flexibility. It is a type of intelligent control system which utilizes the knowledge of system model to achieve better performance. Internal Model Control [IMC] is the most basic form of Model Based Control. This technique is better in set point tracking and disturbance rejection. Today controllers are applied in process industries for controlling pressure, flow, temperature, level etc.

IMC is based on internal model principle which says that is a model of process is known then theoretically perfect control could be achieved. The main advantage of using IMC is that it provides transparent framework for control system design and tuning.

In this thesis we can see that how this IMC structure sometimes reduces to normal feedback structure. For many process it results in a PID controller. The IMC design procedure is just like open loop control design procedure. It gives a freedom in the area of stability as if the process is stable and the controller is stable, then the overall system can't be unstable. This is because the cascaded form of two stable systems will always be stable. The same can't be said for conventional PID controllers as the controlled system could be unstable even if process and controller are individually stable. Being a closed loop system IMC can compensate for disturbances and model uncertainty while open loop systems can't.

In IMC the main concern is set point tracking, but it doesn't guarantees a better disturbance rejection. So in order to achieve good disturbance rejection, an improvement in the controller is designed in which a high pass filter with time constant of the order of disturbance time constant is cascaded to compensate disturbance.

2.2.2 Transfer functions

The representation of output for an input of a single loop feedback system is called the transfer function, which have the transmission in the forward direction from the input to the output. The transfer function among the input d(s) and set-point r(s) and also the process output y(s) is given in Figure (2.3).

The open loop transfer function for Figure 2.1 is given as

$$\frac{y(s)}{r(s)} = g_p(s)q(s) \tag{2.1}$$

The transfer functions for the Figure 2.3 are given as:

$$\frac{y(s)}{r(s)} = \frac{g_p(s)q(s)}{1+q(s)[g_p(s)-g_{pm}(s)]} \tag{2.2}$$

$$\frac{y(s)}{d(s)} = \frac{1-g_{pm}(s)q(s)}{1+q(s)[g_p(s)-g_{pm}(s)]} \tag{2.3}$$

If the modeling is done perfectly, that means

$$g_p(s) = g_{pm}(s) \tag{2.4}$$

Substituting equation 2.3 in 2.2 and 2.1, we get

$$\frac{y(s)}{r(s)} = g_p(s)q(s) \tag{2.5}$$

And

$$\frac{y(s)}{d(s)} = 0 \tag{2.6}$$

From equation 2.5 it is clear that for perfect modeling, the closed loop transfer function reduces to simple open loop system transfer function given by equation 2.1 and it also points towards the primary disadvantage of IMC that is it does not guarantee stability of systems that are open loop unstable. Equation 2.6 shows a perfect disturbance rejection in case of perfect model.

2.2.3 Non Offset Property of IMC

For perfect model, the equation 2.5 is resulted. Now for the output to track the input, the equation changes to

$$\frac{y(s)}{r(s)} = 1 \tag{2.7}$$

Or

$$g_p(s)q(s) = 1 \tag{2.8}$$

Reordering equation 2.8

$$q(s) = {g_p}^{-1}(s) \tag{2.9}$$

Now equation 2.9 concludes that controller should be inverse of process.

2.3 DESIGN PROCEDURE OF INTERNAL MODEL CONTROL

In IMC design procedure we make an assumption that the model is perfect. Model uncertainty is handled by adjusting filter coefficient for robustness and speed of response. It consists of following four steps:

1- Model is divided into two parts, first is invertible other is non-invertible

$$g_{pm}(s) = g_{pm^+}(s)g_{pm^-}(s) \tag{2.10}$$

Where,

$g_{pm}(s)$ is process model

$g_{pm^+}(s)$ is non-invertible part

$g_{pm^-}(s)$ is invertible part

2- Design the ideal IMC controller which is inverse of the invertible part of the process model

$$q'(s) = g_{pm^-}(s) \qquad 2.11$$

3- Now a transfer function is proper only if the order of denominator is at least equal to numerator. So, to make it proper add a filter.

$$q(s) = q'(s)f(s) = g_{p^{-1}}(s)f(s) \qquad 2.12$$

The filter is usually of the form

$$f(s) = \frac{1}{(\lambda s+1)^n} \qquad 2.13$$

4- Now the filter coefficient λ is adjusted to vary the performance of closed loop system between speed of response and robustness.

It should be noted that factorization of transfer function into invertible and non-invertible part is done only for controller design. The process model remains the same full model without any factorization. Also if the process shows inverse response, then closed loop system should also exhibit it and any presence of dead time in the system must also appear in the closed loop response.

2.4 Experimental Setup and Modeling

2.4.1 Experimental Setup

Processes may be classified as linear and non-linear processes. Level Controlling comes under non-linear process and is prone to dominant secondary disturbances. Here the experimental setup consists of four tank system connected to monitoring and controlling platform through DAQ. A schematic diagram of the setup is shown in Figure 2.4.

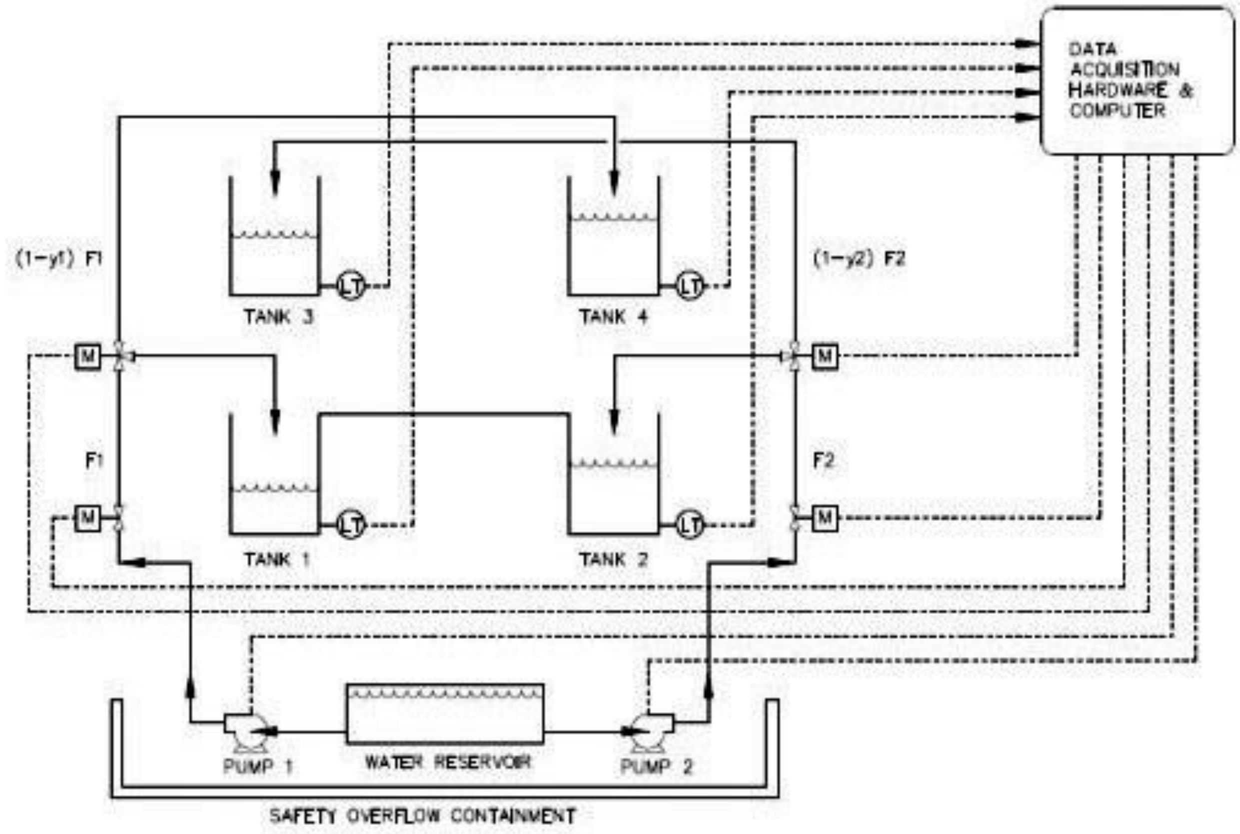

Figure 2.4 Four Tank System

2.4.2 Modeling

2.4.2.1 Modeling of Single Tank System

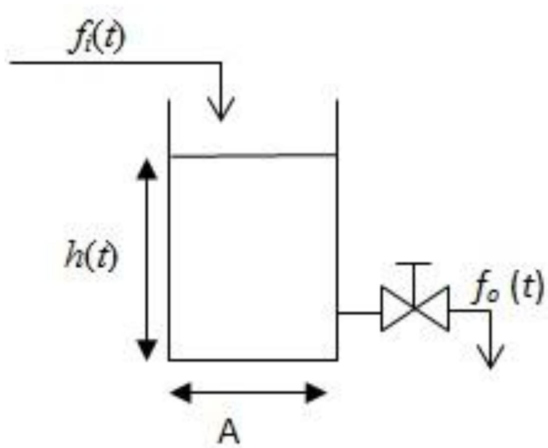

Figure 2.5 Single Tank System

Mass balance equation for tank 1 is given by

$$f_i - f_0 = A\frac{dh}{dt} \tag{2.14}$$

Outflow through the valve is expressed as

$$f_0 = K\sqrt{h} \tag{2.15}$$

f_i is input flow

f_o is output flow through valve

A is area of tank

h is height of liquid in the tank

Linearized equation for the f_o across steady state height h_s is given by

$$A\frac{dh}{dt} = f_i - K[\sqrt{h_s} + 0.5\frac{h-h_s}{\sqrt{h_s}}] \tag{2.16}$$

At steady state

$$A\frac{dh_s}{dt} = f_{is} - K\sqrt{h_s} \tag{2.17}$$

Subtracting equation 2.17 from equation 2.18 we get

$$A\frac{dH}{dt} = F_i - 0.5K\frac{H}{\sqrt{h_s}} \tag{2.18}$$

Where:

$H = h_i – h_s$

$F_i = f_i – f_{is}$

Taking Laplace transform of equation 2.18 we get

$$\frac{\boldsymbol{H(s)}}{\boldsymbol{F_i(s)}} = \frac{\boldsymbol{R}}{\boldsymbol{\tau s+1}} \tag{2.19}$$

Where,

Resistance, $\mathrm{R} = 2\frac{\sqrt{\mathrm{h_s}}}{\mathrm{K}}$

Time Constant, $\tau = 2\mathrm{A}\frac{\sqrt{\mathrm{h_s}}}{\mathrm{K}}$

2.4.2.2 Modeling of Double Tank Non-Interacting System

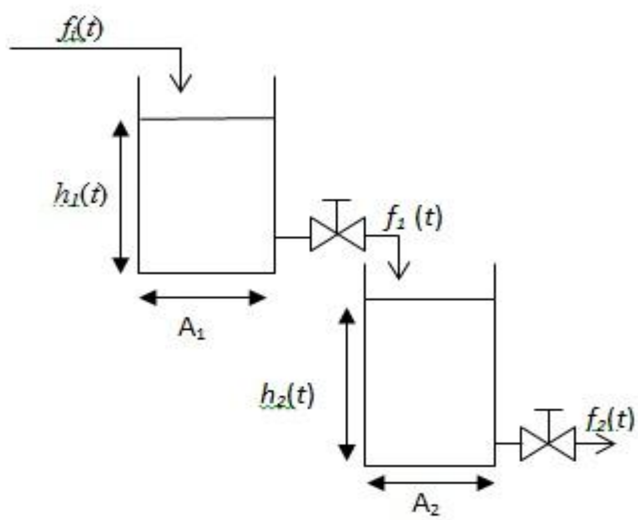

Figure 2.6 Double Tank Non-Interacting System

From equation 2.19, transfer function for the first tank can be given by

$$\frac{H_1(s)}{F_i(s)} = \frac{R_1}{\tau_1 s+1} \tag{2.20}$$

Similarly, for the second tank transfer function can be given by

$$\frac{H_2(s)}{F_1(s)} = \frac{R_2}{\tau_2 s+1} \tag{2.21}$$

Solving equation 2.21 and 2.20 we get

$$\frac{H_2(s)}{F_i(s)} = \frac{R_2}{(\tau_1 s+1)(\tau_2 s+1)} \qquad 2.22$$

2.4.2.2 Modeling of Double Tank Interacting System

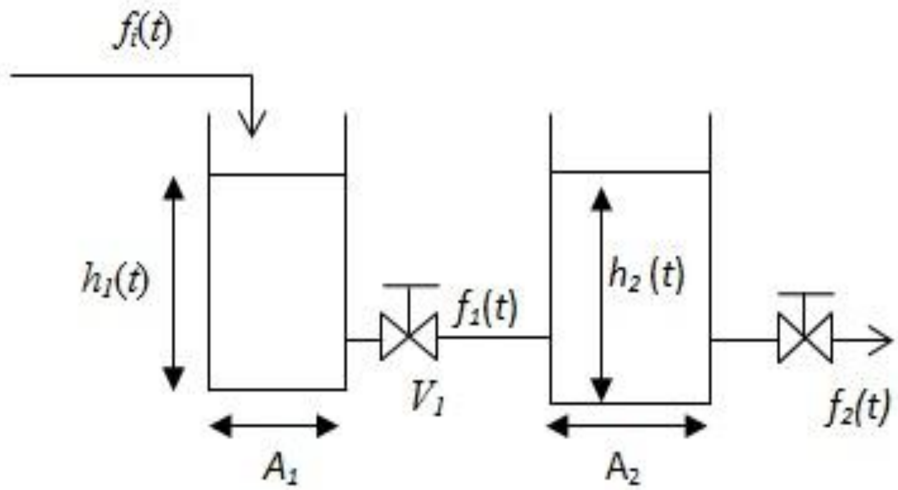

Figure 2.7 Double Tank Interacting System

Mass balance equation for both tanks are given as

$$F_i - F_1 = A_1 \frac{dH_1}{dt} \qquad 2.23$$

$$F_1 - F_2 = A_2 \frac{dH_2}{dt} \qquad 2.24$$

$$F_1 = \frac{H_1 - H_2}{R_1} \qquad 2.25$$

$$F_2 = \frac{H_2}{R_2} \qquad 2.26$$

Solving equations 2.23, 2.24, 2.25 and 2.26 we get transfer function for the two tank interacting system

$$\frac{H_2(s)}{F_i(s)} = \frac{R_2}{(\tau_1 \tau_2 s^2 + \tau_1 s + \tau_2 s + A_1 R_2 s + 1)} \qquad 2.27$$

2.5 OBSERVATIOS, TUNING AND SIMULATION RESULTS

2.5.1 Observations

Process Constants:

1. Manipulated variable is restricted to 200 cm^3/sec.
2. Overshoot height is restricted to 55 cm.

The flow rate and corresponding height observations are given in table 2.1

Flow in lph	Level Transmitter	Height (cm)
350	19.9	49.7
330	17	40.9
310	14.6	32.8
290	12.2	26
270	10.2	19.2
250	8.3	13.7
230	6.5	7.8
210	4.8	2.8
190	4.2	1
170	4.0	0

Table 2.1 Flow and Height Observations

From the observations from the table 2.1, the obtained transfer functions are given in table 2.2.

Flow in lph	Height (cm)	Transfer Functions
350	49.7	$H(s) = \frac{1.898}{335s + 1}$
330	40.9	$H(s) = \frac{1.788}{307s + 1}$
310	32.8	$H(s) = \frac{1.572}{277s + 1}$
290	26	$H(s) = \frac{1.415}{250s + 1}$
270	19.2	$H(s) = \frac{1.245}{220s + 1}$
250	13.7	$H(s) = \frac{1.059}{187s + 1}$
230	7.8	$H(s) = \frac{0.82}{145s + 1}$
210	2.8	$H(s) = \frac{0.516}{92s + 1}$
190	1	$H(s) = \frac{0.32}{56s + 1}$

Table 2.2 Obtained Transfer Functions

Physical parameters of plant are given in table 2.3

Area of tank	176.71 cm^2
Total vertical Height of tank	55cm
Flow Range	20-200cm^3/sec

Table 2.3 Plant Parameters

2.5.2 Tuning

In this section, tuning the controller for IMC has been described. Unlike conventional controllers like P, PI and PID, IMC doesn't cause instability so unlike conventional tuning methods like Ziegler-Nicolas. Hence no stability condition needs to be checked. So the tuning becomes a function of system constants.

We use set point tracking response and disturbance rejection response to choose optimal tuning parameter. Beside these two responses the manipulated variable limit and tank overflow condition is continuously monitored for different filter coefficients. The value of filter coefficient at which the system constants are at the verge of violation is taken as optimal tuning parameter.

Tuning steps for IMC

Step 1: Choose a random value of filter coefficient.

Step 2: Monitor manipulated variable and set point response.

Step 3: If system constants are not violated, decrease the filter coefficient till the verge of constant violation.

Step 4: Select that value of filter coefficient or slightly higher than that.

Now, in order to track the violation point step size needs to be large in the beginning and small later.

2.5.3 Simulation Results

For height range of 40-50 cm, the transfer functions obtained are

For two tank interacting system

$$\frac{H_2(s)}{F_i(s)} = \frac{1.788}{94249s^2+921s+1} \qquad 2.29$$

For two tank non- interacting system

$$\frac{H_2(s)}{F_i(s)} = \frac{1.788}{94249s^2+614s+1} \qquad 2.30$$

2.5.3.1 Two Interacting Tanks

In the first case of double tank interacting system, controller transfer function is given by equation 2.31

$$G_c(s) = .56\,\frac{94000s^2+900s+1}{(\lambda s+1)^2} \qquad 2.31$$

Small mismatch is taken between model and process to show the effect of imperfect modeling.
In order to study the effect of filter coefficient variation over performance, four random values were chosen to be in increasing order. After this different performance indices were analyzed and compare for all four values to choose the best value of tuning parameter.

The observations from the simulation are given in table 2.3 which clearly shows the effect of filter coefficient upon performance. It can be seen that when filter coefficient is small, the system is quicker in compare to higher value of filter coefficient but it becomes more robust for the higher values of tuning parameter.

	λ=30	λ=70	λ=100	λ=130
Rise Time(sec)	123	200	263	315
Settling time(sec)	1412	1978	2215	2375
Percentage Overshoot	10.92	12.56	15.7	17.8
Peak time(sec)	303	543	669	784

Table 2.3 Comparison of performance index for different filter coefficients in IMC strategy for interacting tank system

The results shown in Table 2.3 are obtained from Figure 2.8 which shows the set point tracking with varying filter coefficients for IMC. The different values that have been taken are λ=30, 70, 100 and 130.

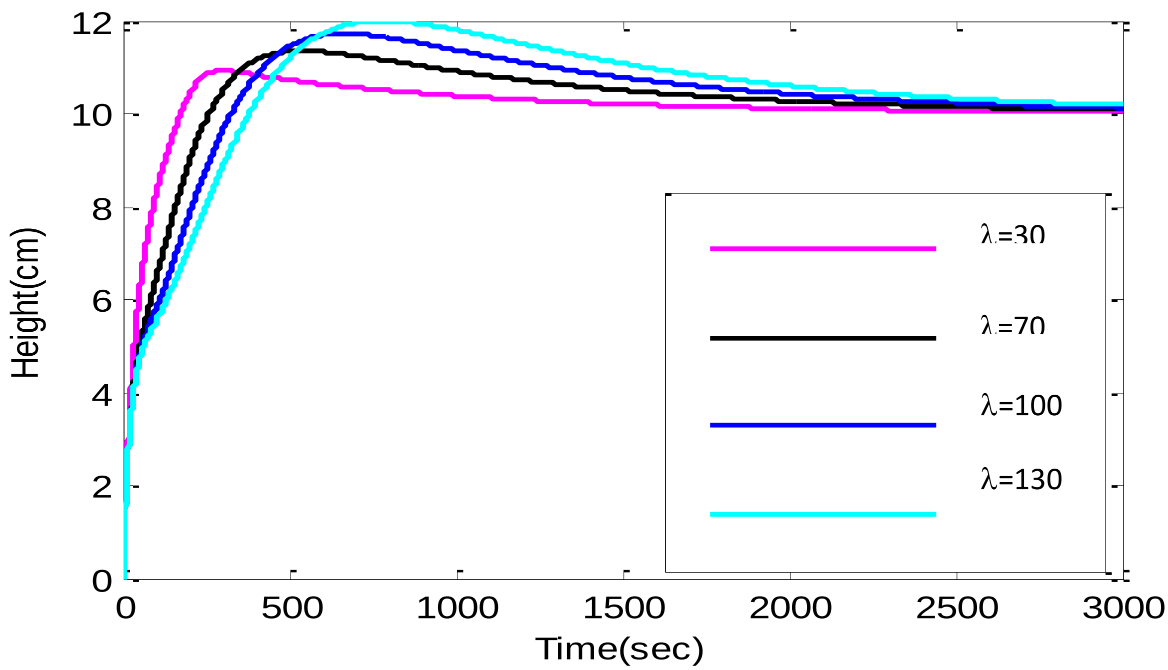

Figure 2.8 Set point tracking response with IMC for the interacting tank system

Figure 2.9 shows the disturbance rejection response of double tank interacting system with added primary and secondary disturbance for IMC. Here both disturbances are of same value.

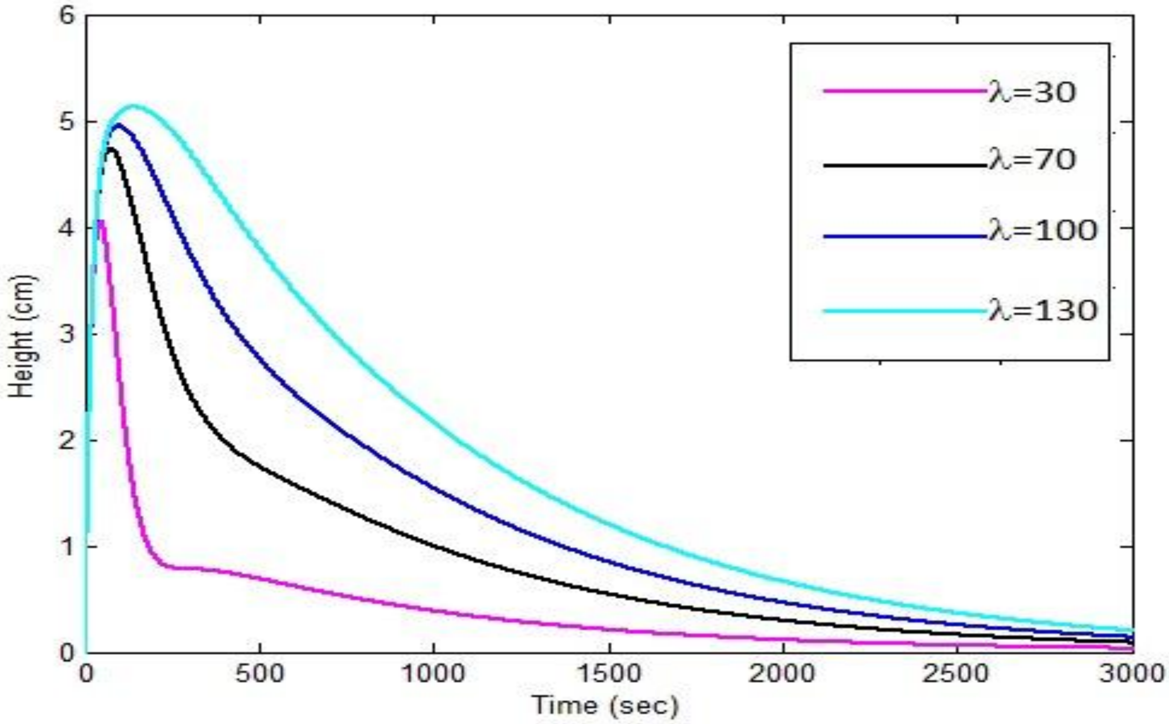

Figure 2.9 Disturbance Rejection response with IMC for the interacting tank system

IMC with improved disturbance rejection

Improvement in disturbance rejection can be achieved by slight variation in the filter. This is achieved by adding a high pass filter with a factor ‘γ’, which is selected so as to cancel slow disturbance time constant. The filter now becomes of the form

$$f(s) = \frac{\gamma s+1}{(\lambda s+1)^n} \qquad 2.33$$

Where,

f(s) is filter transfer function

λ is filter coefficient

γ is correction factor

n is order of filter

The output in case of perfect model now becomes

$$y(s) = [\frac{(\lambda s+1)^n - g_{pm^+}(s)(\gamma s+1)}{(\lambda s+1)^n}] g_d(s)d(s) \qquad 2.34$$

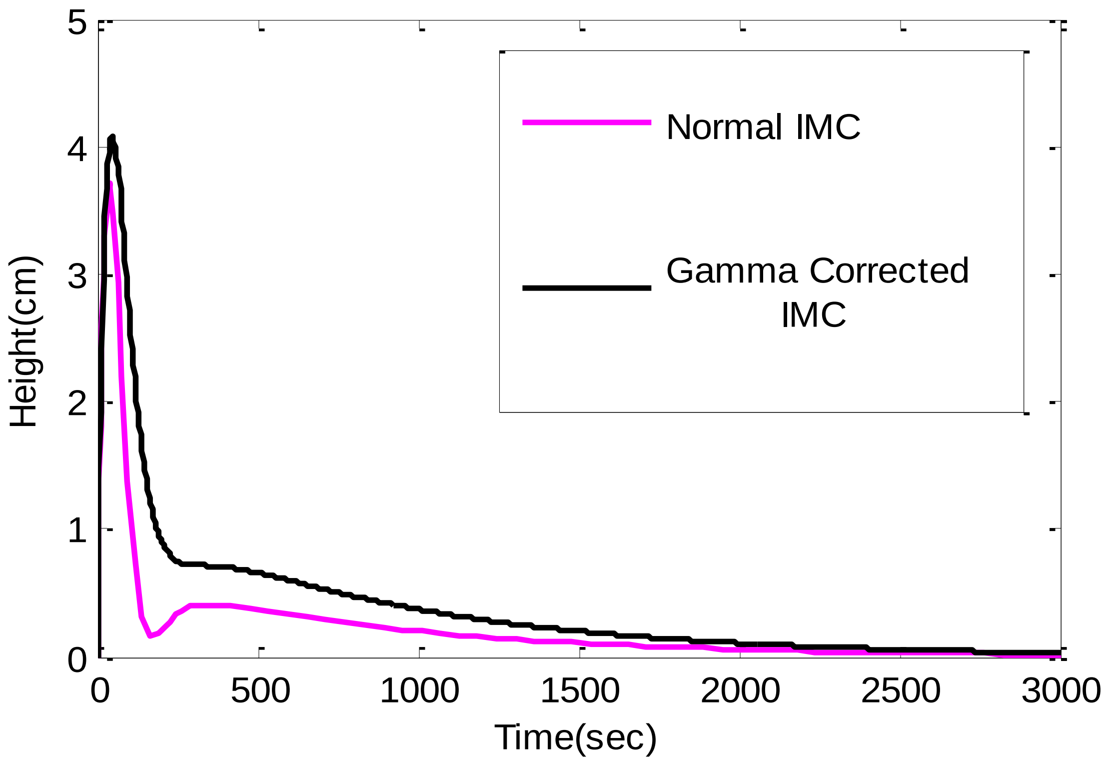

Figure 2.10 Improved Disturbance Rejection response with IMC for the interacting tank system

This response is obtained by using above transfer function for two interacting tanks and controller transfer function is

$$G_c(s) = .56\frac{94000s^2+900s+1}{(70s+1)^2} \qquad 2.32$$

Here filter coefficient is taken as λ=70.

A precaution should be taken that introduction of correction factor in filter could alter the response such that it violates system constants.

2.5.3.2 Two Non-Interacting Tanks

In the first case of double tank interacting system, controller transfer function is given by equation 2.31

$$G_c(s) = .56\,\frac{94000s^2+600s+1}{(\lambda s+1)^2} \qquad 2.35$$

The same procedure is applied for non-interacting tanks also to get the set point tracking as well as disturbance rejection by varying the value of filter coefficient. Model uncertainty approximation is also considered here.

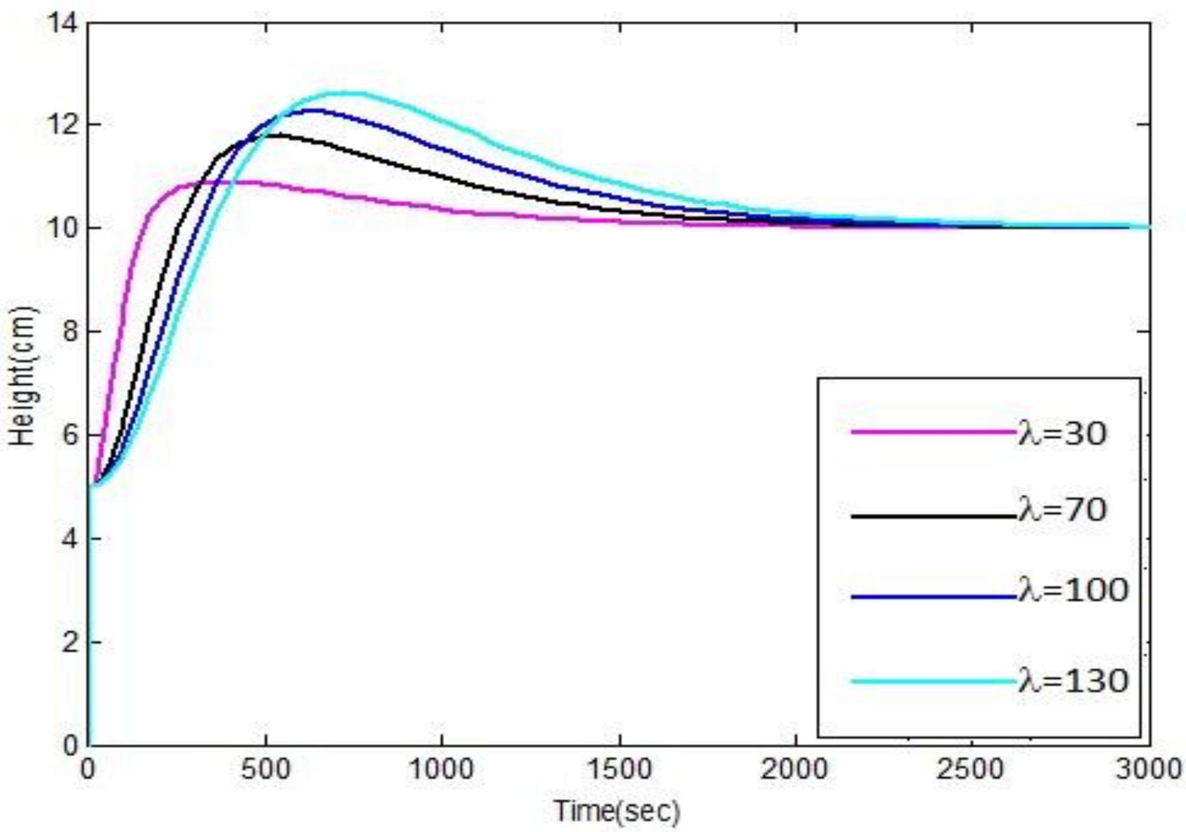

Figure 2.11 Set point tracking response with IMC for the non-interacting tank system

The open loop interacting system is sluggish than that of non-interacting system but closed loop IMC controlled responses for both the systems is nearly same.

	λ=30	λ=70	λ=100	λ=130
Rise Time(sec)	114	204	253	290
Settling time(sec)	1272	1711	1932	2114
Percentage Overshoot	8.89	17.95	22.66	25.89
Peak time(sec)	386	525	628	718

Table 2.4 Comparison of performance index for different filter coefficients in IMC strategy for non-interacting tank system

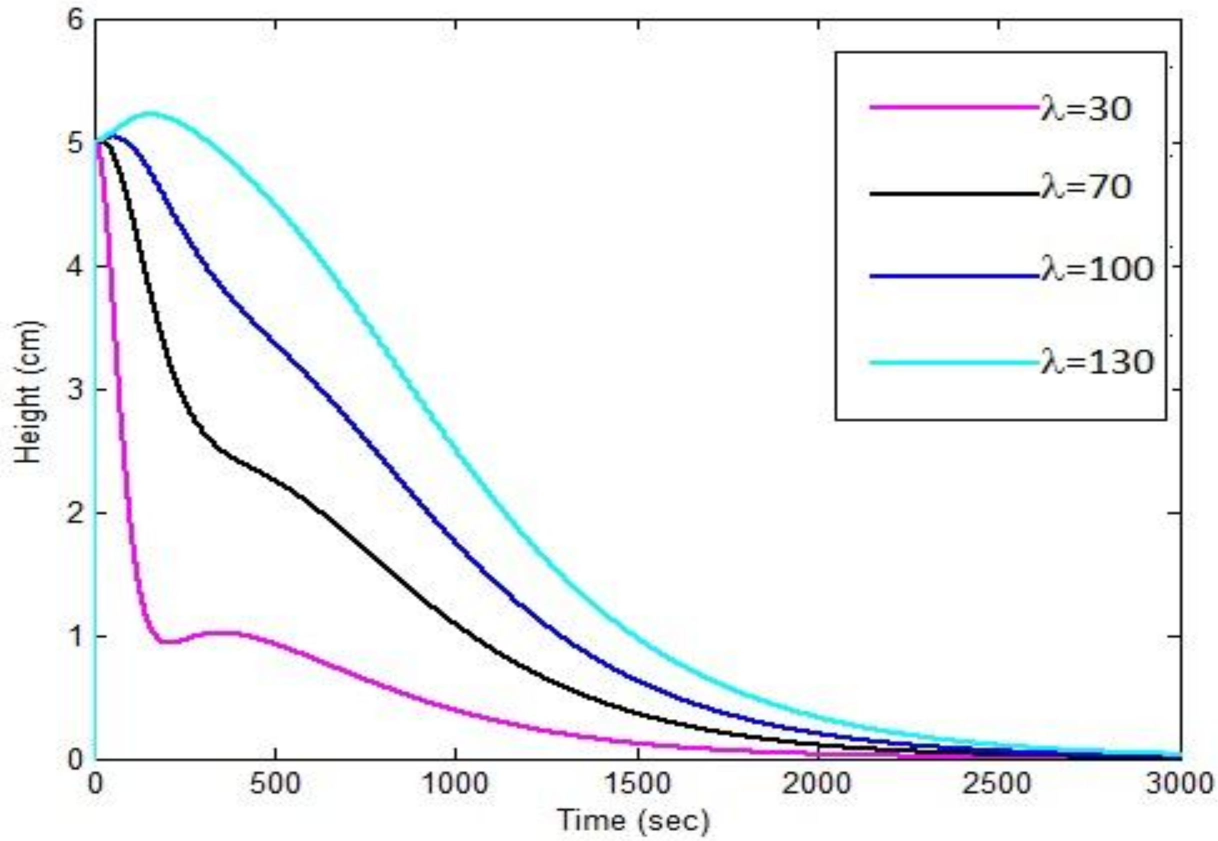

Figure 2.12 Disturbance Rejection response with IMC for the non-interacting tank system

CHAPTER-3

IMCC SYSTEM

3.1 INTRODUCTION

3.2 CASCADE STRUCTURE AND CONTROLLER DESIGN

3.3 SIMULATION RESULTS AND DISCUSSIONS

3.1 INTRODUCTION

3.1.1 Cascade Control System

Cascade Control is one of the popular control strategies. Until now we have focused mainly on the set point tracking for the system. It is easier to tune for set point changes as we don't know when the disturbance is going to play its role. In practical system like in multi tank level control system, disturbance rejection plays a very important role. The major disadvantage in conventional controllers is that the disturbance must be first reflected into output variable before it could be controlled. Here in cascade control, the objective is to compensate the disturbance before it could be felt in primary variable. Cascade strategy is basically used in the following two conditions:

1. Disturbance is directly affecting the secondary output or intermediate variable which in result affects the primary output variable that needs to be controlled.

2. The gain of secondary process is non-linear.

In the first case cascade system would reduce the effect of disturbance entering in secondary process on the primary output. In second case, the effect of non-linear gain of secondary variable on the primary variable can be limited.

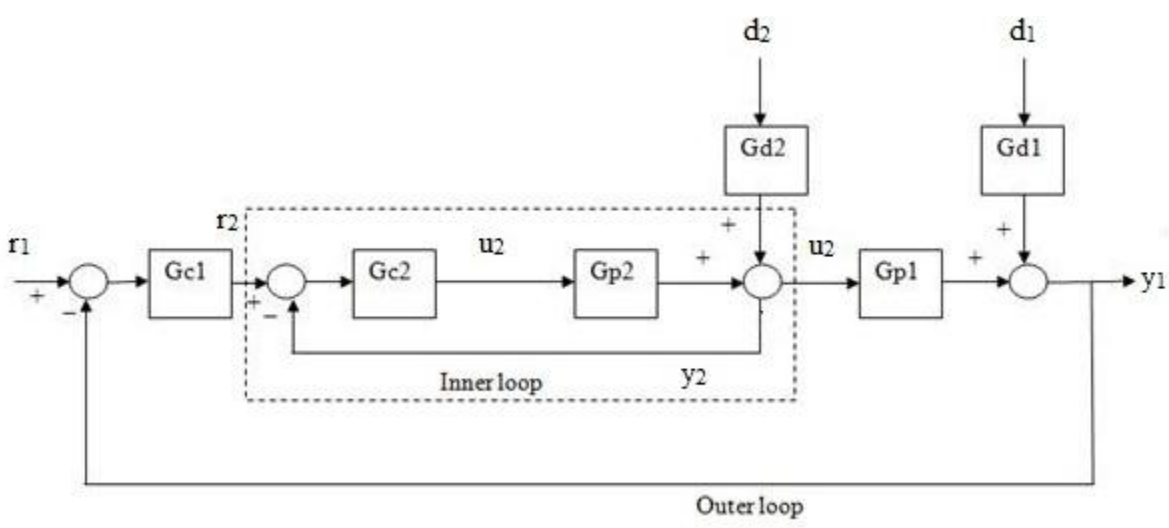

Figure 3.1 Block diagram of Cascade System

Where,

Gc_1 - Primary Controller

Gc_2 - Secondary Controller

Gp_1 - Primary Process

Gp_2 - Secondary Process

Gd_1 - Primary Disturbance Gain

Gd_2 - Secondary Disturbance Gain

d_2 - Secondary Disturbance

d_1 - Primary Disturbance

y_1 - Primary Output

y_2 - Secondary output

r_1 - Primary Set-point

r_2 - Secondary Set-point

3.1.2 System Transfer Function

The secondary output is given by

$$y_2(s) = \frac{G_{c2}(s)G_{p2}(s)}{1+G_{c2}(s)G_{p2}(s)} r_2(s) + \frac{G_{d2}(s)}{1+G_{c2}(s)G_{p2}(s)} d_2(s) \qquad 4.1$$

The secondary closed loop transfer function can be defined as

$$G_{c2cl} = \frac{G_{c2}(s)G_{p2}(s)}{1+G_{c2}(s)G_{p2}(s)} \qquad 4.2$$

The primary output of the system is

$$y_1(s) = \frac{1+G_{c2}(s)G_{p2}(s)G_{p1}(s)}{1+G_{c2}(s)G_{p2}(s)} r_2(s) + \frac{G_{d2}(s)G_{p1}(s)}{1+G_{c2}(s)G_{p2}(s)} d_2(s) + d_1(s)G_{d1}(s) \qquad 4.3$$

After tuning the inner loop, the following transfer function can be used to design the outer controller

$$G_{c1eff}(s) = \frac{G_{c2}(s)G_{p2}(s)G_{p1}(s)}{1+G_{c2}(s)G_{p2}(s)} = G_{c2cl}(s)G_{p1}(s) \qquad 4.4$$

The output for primary set point change can be given as

$$y_1(s) = \frac{G_{c1}(s)G_{p1eff}(s)}{1+G_{c1}(s)G_{p1eff}(s)} r_1(s) = \frac{G_{c1}(s)G_{c2cl}(s)G_{p1}(s)}{1+G_{c1}(s)G_{c2cl}(s)G_{p1}(s)} r_1(s) \qquad 4.5$$

Here, the secondary loop is made much faster by using a high gain proportional controller, thus making

$G_{c2cl}(s)=1$

The secondary loop is faster with respect to primary loop speed. If primary loop is much slow a low gain controller could also make secondary loop much faster than that if the primary loop. Thus compensating errors before it could affect the primary output.

Using the unity value of G_{c2cl} in equation 4.5 we get

$$y_1(s) \sim \frac{G_{c1}(s)G_{p1}(s)}{1+G_{c1}(s)G_{p2}(s)} \qquad 4.6$$

The approximation symbol is used rather than using equality to specify the condition that practically

$G_{c2cl}(s) \neq 1$

meaning secondary loop can't be made infinitely faster.

3.2 CASCADE STRUCTURE AND CONTROLLER DESIGN

3.2.1 Cascade Structure

The traditional cascade strategy consists of two PID controllers as secondary and primary controllers. The secondary controller is generally taken to be high gain proportional controller in order to make the secondary loop faster as compared to primary loop. The disturbance d_2 is compensated by the fast secondary loop and reduces its effect on the primary variable. Figure 3.2 shows a conventional cascade strategy using PID controllers.

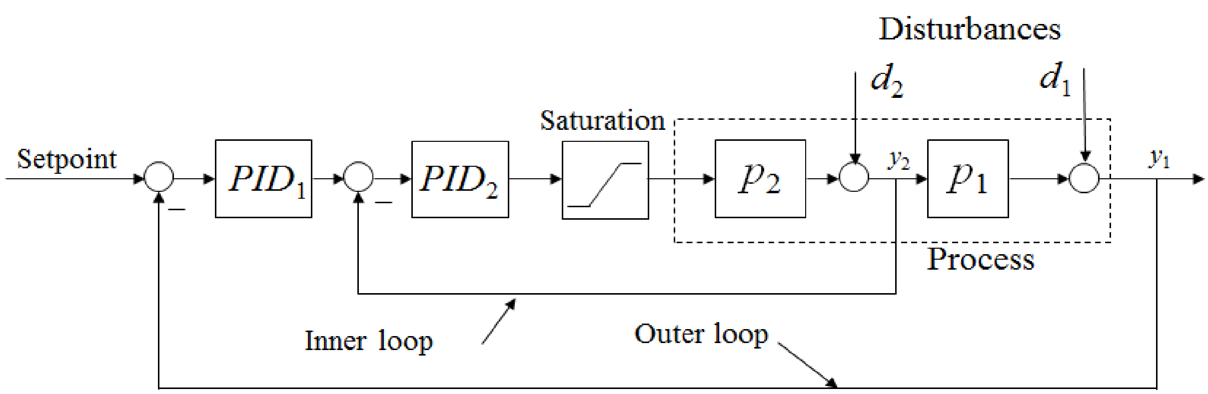

Figure 3.2 Block diagram of Traditional Cascade System

The saturation block in the above block diagram shows the operating limits of control valve as the high gain secondary controller could lead to saturation of the manipulated variable.

In IMCC both of the PID controllers are replaced by IMC. Having IMC it needs to have models of both primary as well as secondary process. It utilizes advantage of both IMC and Cascade control to give both good set point tracking as well as disturbance rejection. In IMCC the secondary controller needs to be tuned just to remove the effect of secondary disturbances. The primary controller follows the same tuning as IMC. Figure 3.3 shows the basic Internal Model Cascade Control Strategy.

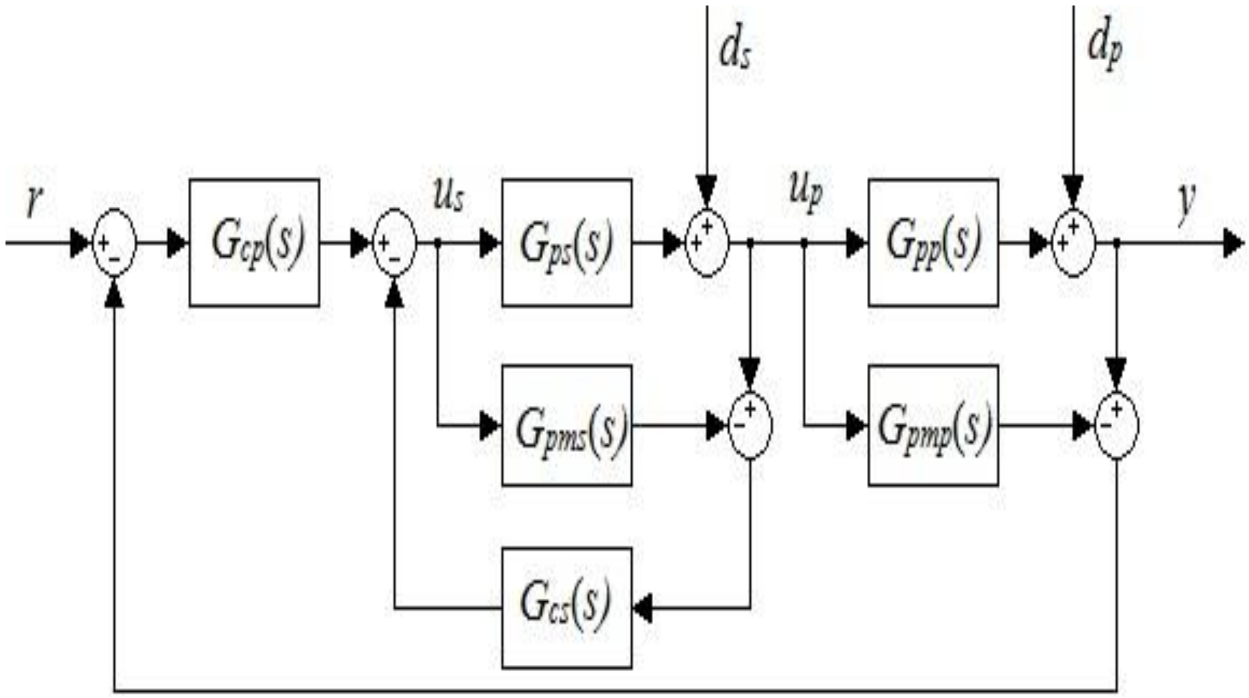

Figure 3.3 Block diagram of IMCC System

In the above diagram,

$G_{cp}(s)$ is primary controller

$G_{cs}(s)$ is secondary controller

$G_{pp}(s)$ is primary process

$G_{ps}(s)$ is secondary process

$G_{pmp}(s)$ is primary process model

$G_{pms}(s)$ is secondary process model

d_p is primary disturbance

d_s is secondary disturbance

u_p is primary manipulated variable

u_s is secondary manipulated variable

r is set point

y is primary output

3.2.2 Controller Design

From the figure 3.3, the secondary process output can be given as

$$y'(s) = \frac{G_{ps}(s)u_s(s)+(1-G_{pms}(s))G_{cs}(s)d_2(s)}{1+(G_{ps}(s)-G_{pms}(s))G_{cs}(s)} \qquad 4.7$$

The transfer function for primary output is

$$y(s) = \frac{G_{pp}(s)G_{ps}(s)r(s)+\left(1-G_{pms}(s)G_{cs}(s)\right)G_{pp}(s)d_2(s)}{1+\left(G_{pp}(s)-G_{pmp}(s)\right)G_{ps}(s)G_{cp}(s)+(G_{ps}(s)-G_{pms}(s))G_{cs}(s)} +$$

$$\frac{(1-G_{pmp}(s)G_{ps}(s)G_{cp}(s)+(G_{ps}(s)-G_{pms}(s))G_{cs}(s))d_1(s)}{1+\left(G_{pp}(s)-G_{pmp}(s)\right)G_{ps}(s)G_{cp}(s)+(G_{ps}(s)-G_{pms}(s))G_{cs}(s)} \qquad 4.8$$

From equation 4.7 it can be said that

1. If the time constant of primary process is larger than that of secondary process the secondary controller should be designed in such a way that zeros of $(1-G_{pms}(s)G_{cs}(s))$ cancel the large time constant of primary process. If the condition is not satisfied then controller design should be as normal IMC.

2. The primary controller should be

$$G_{cp}(s) = {G_{pmp}}^{-1}(s){G_{pms}}^{-1}(s) \qquad 4.9$$

After designing, the tuning of both the controllers needs to be done to determine the optimal tuning parameter values for both primary as well as secondary controller in order to give desired performance.

3.3 CONTROLLER TUNING AND SIMULATION RESULTS

3.3.1 Controller Tuning

The tuning constraints remain the same to be process constants but the number of controllers is two. Tuning steps for IMCC are

Step 1: Open the primary loop.

Step 2: For secondary controller, choose a random value of filter coefficient.

Step 3: Monitor manipulated variable and set point response.

Step 4: If system constants are not violated, decrease the filter coefficient till the verge of constant violation.

Step 5: Select that value of filter coefficient or slightly higher than that.

Step 6: Now close primary loop with secondary loop having the tuned filter coefficient.

Step 7: For primary controller, choose a random value of filter coefficient.

Step 8: Monitor manipulated variable and set point response.

Step 9: If system constants are not violated, decrease the filter coefficient till the verge of constant violation.

Step 10: Select that value of filter coefficient or slightly higher than that.

In case of IMCC improved disturbance rejection could be achieved by using the same method for IMC. But for IMCC, correction factor could be introduced for any one of the filter or both filters. If both filters are enhanced, the above tuning pattern should be followed (first secondary loop while primary loop open, the primary loop while secondary loop closed).

A precaution should be taken that introduction of correction factor in filter could alter the response such that it violates system constants. The form of filter for both controllers remains the same to be

$$f_1(s) = \frac{\gamma_1 s+1}{(\lambda_1 s+1)^n} \qquad 4.10$$

$$f_2(s) = \frac{\gamma_2 s+1}{(\lambda_2 s+1)^n} \qquad 4.11$$

3.3.2 Simulation Results

3.3.2.1 Two Interacting Tanks

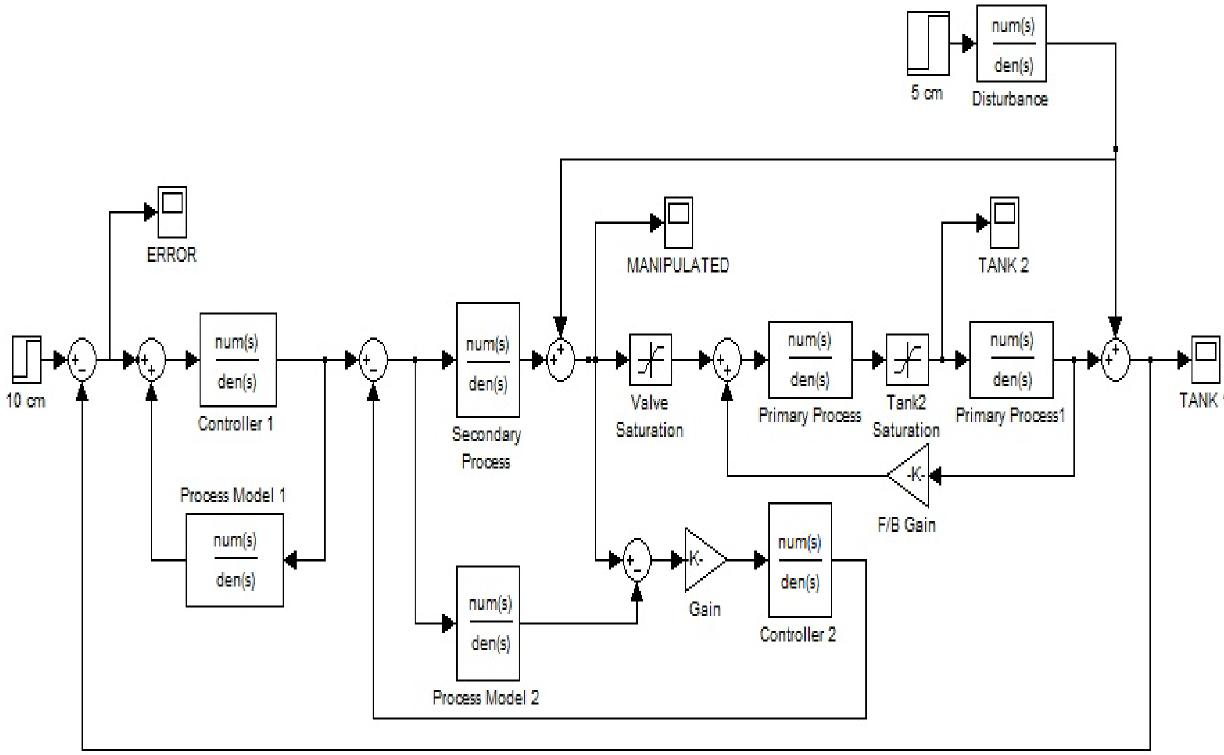

Figure 3.4 SIMULINK diagram of IMCC design for double tank Interacting System

In the above simulation diagram two saturators have been used. The first represents the first process constant i.e. valve saturation, and the second one represents the other system constant i.e. tank height overflow. The feedback in the process loop represents the dependency of one tank height on other tank height, since both tanks are interacting.

The monitoring was done in similar way as in IMC but with a difference that there were two tuning parameters λ_1 and λ_2 for primary and secondary controller respectively. Figure 3.5 shows the simulation results of comparison between different responses with varying tuning parameters. The values of tuning parameter are λ_1 = 30, 70, 100, 130 and λ_2 = 10, 25, 40, 55 paired in respective manner.

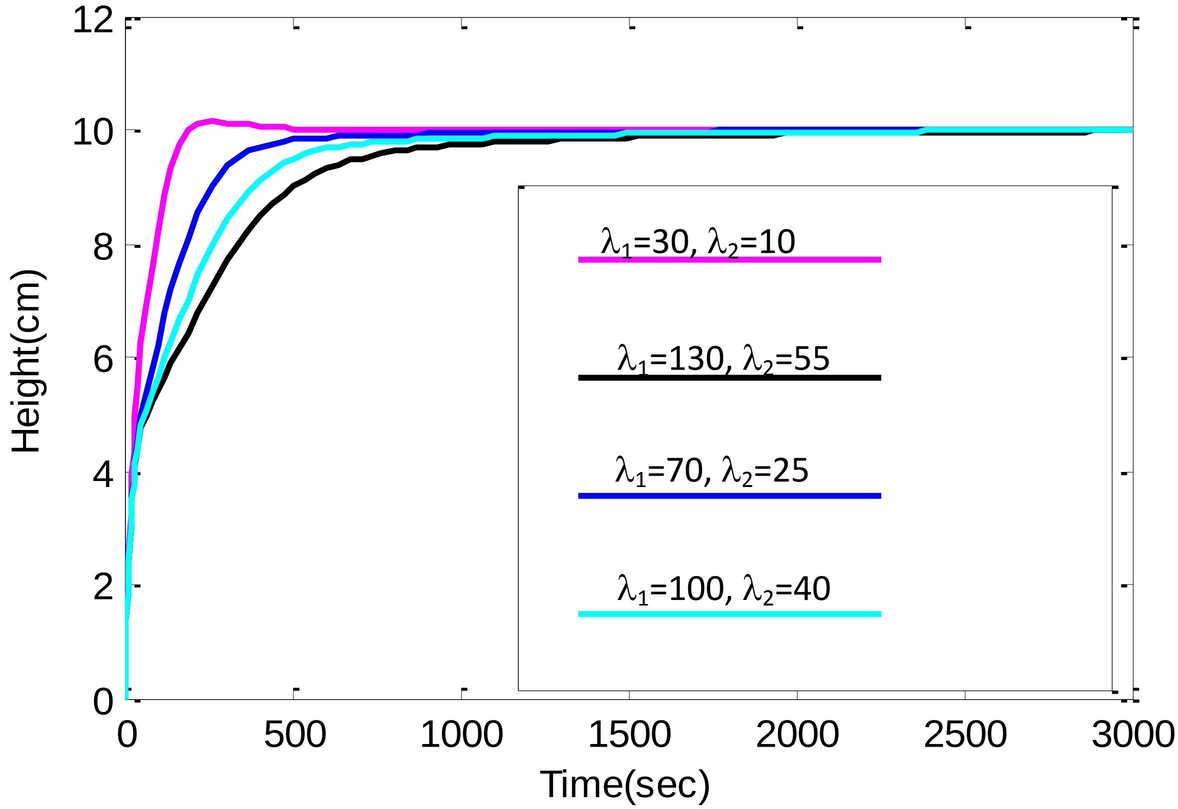

Figure 3.5 Set point tracking with IMCC for two tank Interacting System

Table 3.1 contains the observations obtained from figure 3.4

	λ_1=30 λ_2 =10	λ_1=70 λ_2 =25	λ_1=100 λ_2 = 40	λ_1=130 λ_2 =55
Rise Time(sec)	121	253	373	495
Settling time(sec)	170	467	771	1122
Percentage Overshoot	1.6	0	0	0
Peak Time(sec)	255	---	---	---

Table 3.1 Comparison of performance index for different filter coefficients in IMCC strategy for interacting tank system

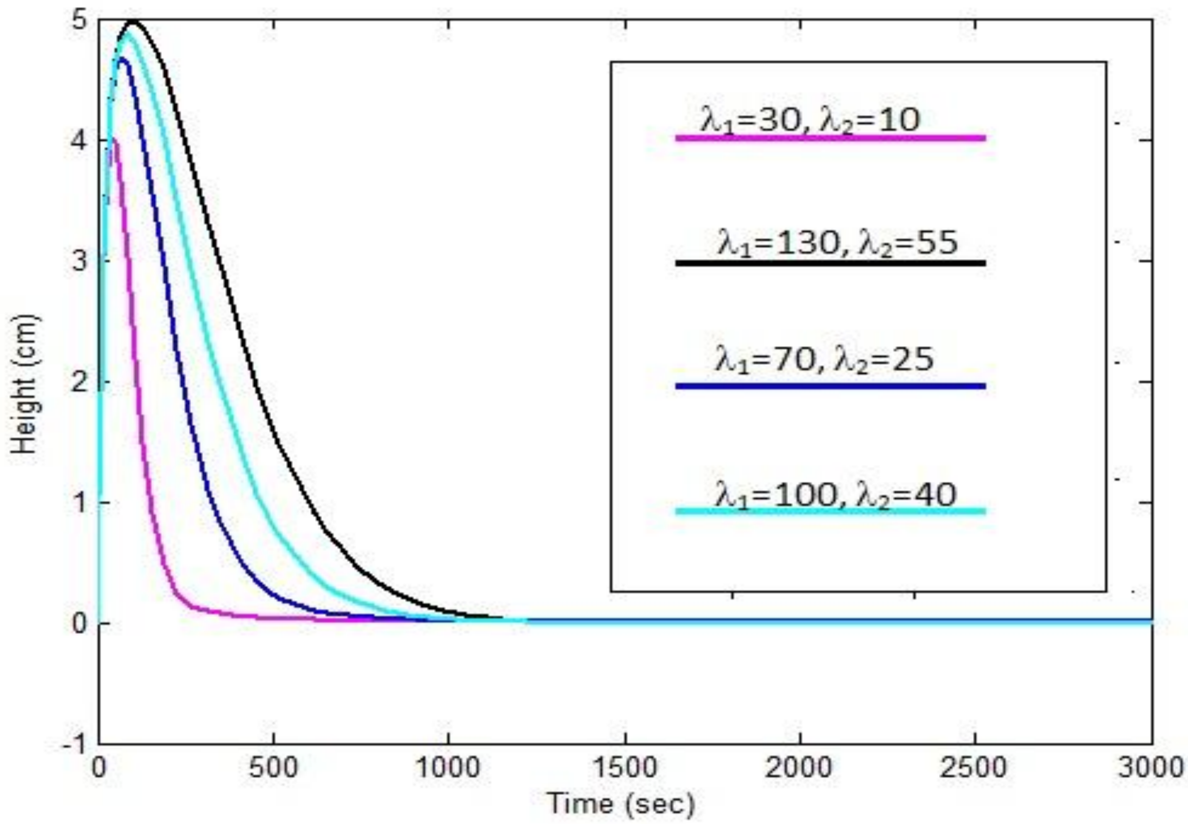

Figure 3.6 Disturbance Rejection with IMCC for two tank Interacting System

From the figure 3.5 it can be clearly seen that decreasing the filter coefficients the transient response of the system is getting improved but decreasing the parameter after certain extent, will also result in the increase in the peak overshoot. For the whole process above the secondary tank level and manipulated variable were constantly monitored to get the optimal value for the tuning parameter.

Figure 3.6 shows the disturbance rejection using IMCC with varying values of primary and secondary filter coefficients. The disturbance rejection can be seen to be improving with decreasing filter coefficients but decease in filter coefficients improves system performance but erodes the robustness of the controller (immunity to modeling errors as well as errors in controller designing). Figure 3.7 shows the comparative results of controller performance for the Interacting Tank process. The controllers that are compared are PID, PID Cascade, IMC and IMCC.

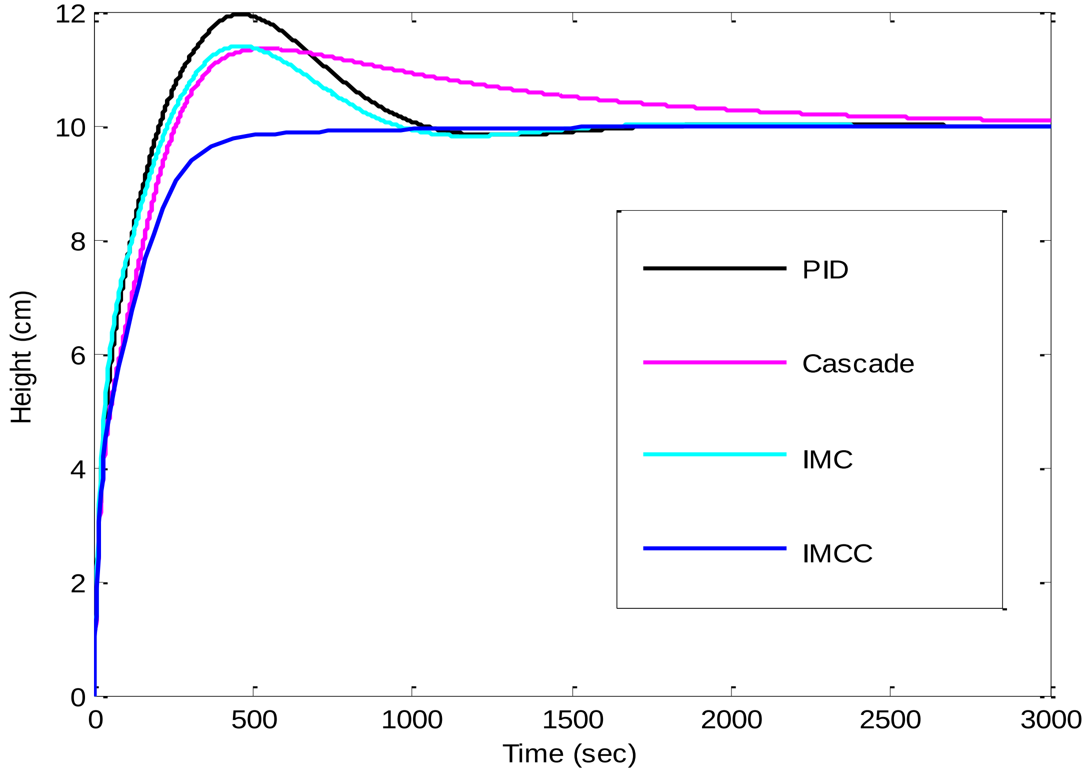

Figure 3.7 Comparison of different controllers for two tank Interacting System

The statistical comparison obtained from the data of figure 3.7 is given in table 3.2.

	PID	Cascade	IMC	IMCC
Rise Time(sec)	153	165	200	253
Settling time(sec)	946	865	1978	467
Percentage Overshoot	19.52	14	12.56	0
Peak time(sec)	467	464	543	----

Table 3.2 Comparison of performance index with different controllers for interacting tank system

3.2.3.1 Two Non-Interacting Tanks

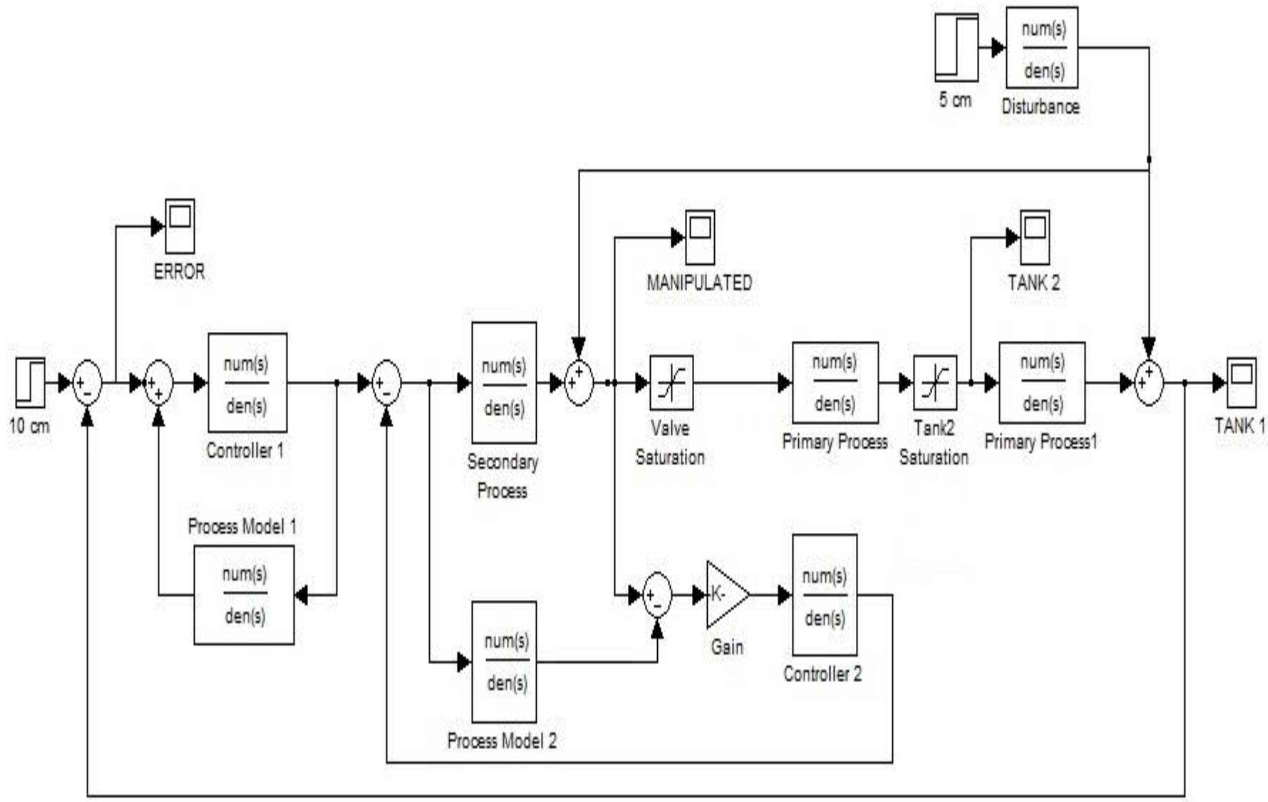

Figure 3.8 SIMULINK diagram of IMCC design for double tank Non-Interacting System

The statistical comparison obtained from the data of figure 3.9 is given in table 3.3.

	λ_1=30 λ_2 =10	λ_1=70 λ_2 =25	λ_1=100 λ_2 = 40	λ_1=130 λ_2 =55
Rise Time(sec)	123	280	397	522
Settling time(sec)	193	571	1076	1422
Percentage Overshoot	0	0	0	0

Table 3.3 Comparison of performance index for different filter coefficients in IMCC strategy for non-interacting tank system

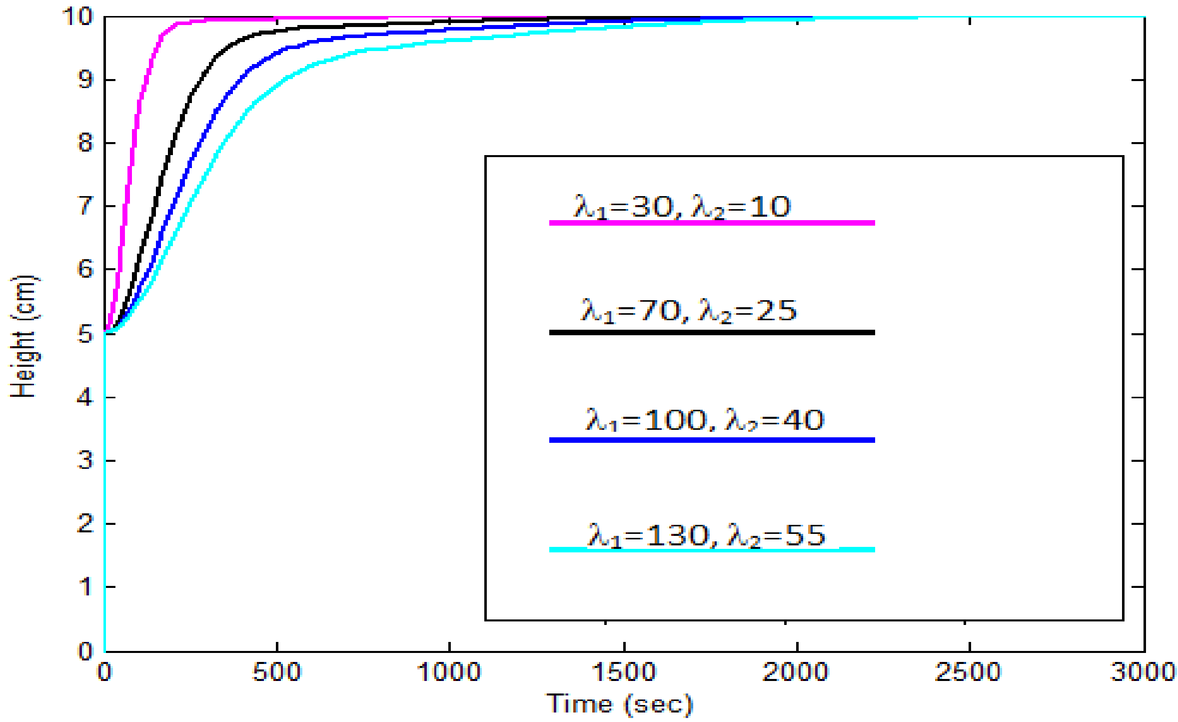

Figure 3.9 Set point tracking with IMCC for two tank Non-Interacting System

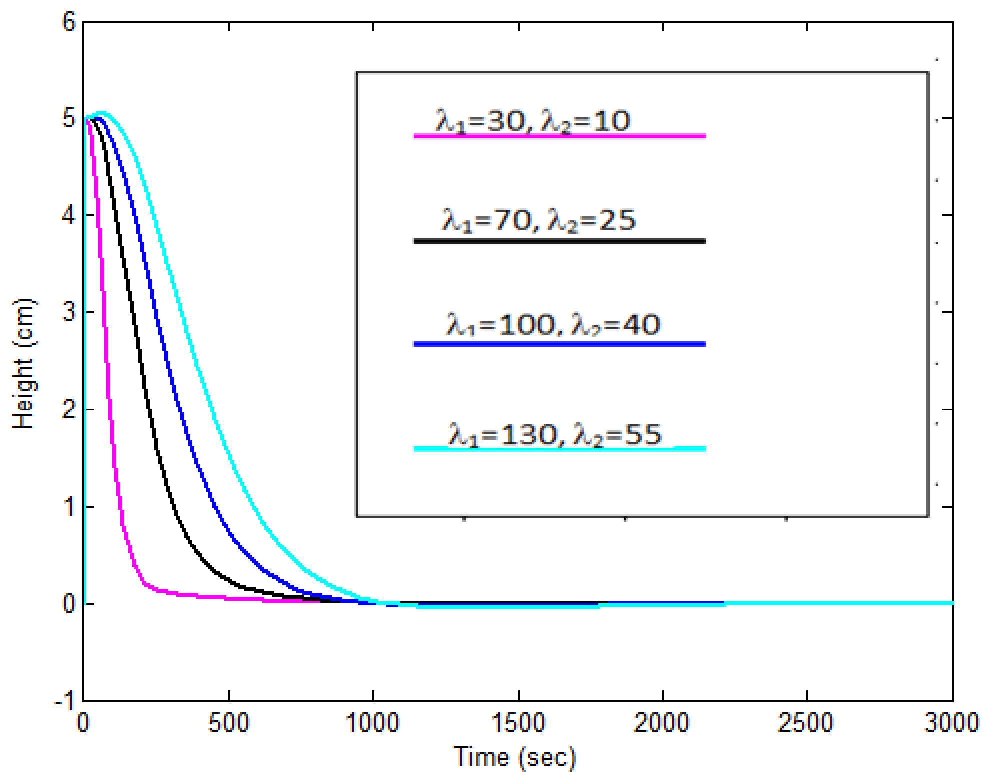

Figure 3.10 Disturbance Rejection with IMCC for two tank Non-Interacting System

CHAPTER-4

IMC BASED PID SYSTEM

4.1 INTRODUCTION

IMC has a property that the controller tuning can be done by using only a single parameter and that is filter coefficient of the controller. The filter coefficient in any system is equivalent to the time constant of the closed loop system. Although IMC procedure is very easy, the most commonly used controller in industries is still PID controller. In this chapter IMC is rearranged to give a standard feedback controller. Here Empirical formula has also been developed for the tuning of controller to give the desired performance meeting all the given performance conditions. IMC can be reordered to give Figure 4.1.

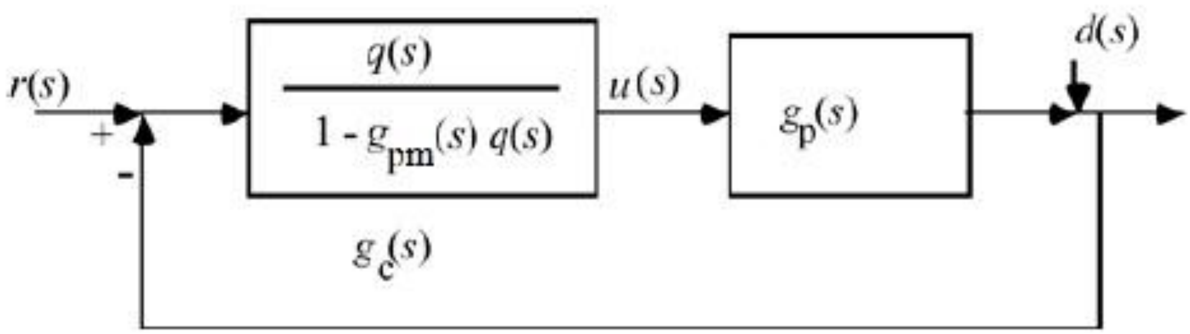

Figure 4.1 Feedback equivalent of IMC

Where,

- q(s) is IMC controller
- $g_{pm}(s)$ is Process model
- $g_c(s)$ is Feedback controller
- $g_p(s)$ is Process

The transfer function for the feedback controller and input-output is given by

$$g_c(s) = \frac{q(s)}{1 - g_{pm}(s) q(s)} \quad 4.1$$

$$\frac{y(s)}{r(s)} = \frac{q(s) g_p(s)}{1 - q(s)(g_{pm}(s) - g_p(s))} \quad 4.2$$

4.2 DESIGN PROCEDURE

Design procedure consists of following steps.

1. Obtain the IMC controller transfer function which has filter cascaded with it in order to make it proper. In order to get improved disturbance rejection, the filter of the form described in chapter 2 should be used.

$$f(s) = \frac{\gamma s+1}{(\lambda s+1)^n} \tag{4.3}$$

2. Get the equivalent PID controller transfer function

$$g_c(s) = \frac{q(s)}{1-g_{pm}(s)q(s)} \tag{4.4}$$

3. Show equation 4.4 in PID form and find k_c, τ_I, τ_D. It sometimes comes directly to standard PID equations with first order filter cascaded with it.

$$g_c(s) = k_c\left[\frac{\tau_I \tau_D s^2+\tau_I s+1}{\tau_I s}\right]\left[\frac{1}{\tau_F s+1}\right] \tag{4.5}$$

τ_F is filter time constant

k_c is proportional gain

τ_D is derivative time constant

τ_I is integral time constant

4. Perform closed loop simulations and adjust λ considering a tradeoff between performance and robustness. Another method for getting best values of λ has been suggested later in this chapter.

4.2.1 Design for the first order process

Process model for the first order process can be given by

$$g_{pm}(s) = \frac{k_p}{\tau_p s+1} \qquad 4.6$$

Now the IMC controller transfer function is obtained

$$q(s) = \frac{1}{k_p}\frac{\tau_p s+1}{\lambda s+1} \qquad 4.7$$

Now equivalent feedback controller using the transformation is given as

$$g_c(s) = \frac{q(s)}{1-g_{pm}(s)q(s)} = \frac{\tau_p s+1}{k_p \lambda s} \qquad 4.8$$

The standard equation for PI controller is given by

$$g_c(s) = k_c \frac{\tau_I s+1}{\tau_I s} \qquad 4.9$$

Equation 4.8 can be rearranged to

$$g_c(s) = \frac{\tau_p}{k_p \lambda}\frac{\tau_p s+1}{\tau_p s} \qquad 4.10$$

Comparing equations 4.9 and 4.10 we get

$k_c = \frac{\tau_p}{k_p \lambda}$ and $\tau_I = \tau_p$

4.2.2Design for the second order process

Process model for second order process can be given as

$$g_{pm}(s) = \frac{k_p}{(\tau_1 s+1)(\tau_2 s+1)} \qquad 4.11$$

Now the IMC controller can be given as

$$q(s) = \frac{\tau_1 s+1}{k_p}\frac{\tau_2 s+1}{\lambda s+1} \qquad 4.12$$

Now equivalent feedback controller using the transformation is given as

$$g_c(s) = \frac{q(s)}{1-g_{pm}(s)q(s)} = \left[\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{k_p\lambda s}\right] \qquad 4.13$$

The standard equation for PID controller is given by

$$g_c(s) = k_c\left[\frac{\tau_I\tau_D s^2+\tau_I s+1}{\tau_I s}\right] \qquad 4.14$$

Equation 4.13 can be rearranged to

$$g_c(s) = \left[\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{(\tau_1+\tau_2)s}\right]\frac{(\tau_1+\tau_2)}{k_p\lambda} \qquad 4.15$$

Comparing equation 4.14 and 4.15 we get

$$k_c = \frac{(\tau_1+\tau_2)}{k_p\lambda} \qquad 4.16$$

$$\tau_I = \tau_1 + \tau_2 \qquad 4.17$$

$$\tau_D = \frac{(\tau_1\tau_2)}{(\tau_1+\tau_2)} \qquad 4.18$$

4.3 EMPIRICAL FORMULA AND SIMULATION RESULTS

The IMC has been designed based on PID procedure. The transfer function for the single tank is used, which is

$$g_p(s) = \frac{k_p}{\tau_p s+1}$$

The simulation results of Rise Time and Settling Time with the variation in filter coefficient and process time constant are shown in below table.

	τ_p=200		τ_p=225		τ_p=250		τ_p=275		τ_p=300	
Filter Coefficient λ	t_r	t_s	t_r	t_s	t_r	t_s	t_r	t_s	t_r	t_s
	sec	sec	sec	sec	sec	sec	sec	sec	sec	sec
30	44	80	50	90	55	100	60	110	66	120
40	58	106	66	119	73	132	80	145	88	159
50	73	132	82	148	91	165	100	181	110	198
60	88	158	98	178	110	198	121	217	132	237
70	102	184	115	207	128	230	141	253	154	276
80	117	211	132	237	146	263	161	289	176	316
90	132	237	148	266	165	296	181	324	198	354
100	146	263	164	295	183	328	201	360	219	393
110	161	289	181	324	201	360	221	396	242	432
120	176	315	198	354	219	393	241	432	264	472
130	190	341	214	383	238	426	261	468	286	510

	τ_p=325		τ_p=350		τ_p=375		τ_p=400		τ_p=425	
Filter Coefficient λ	t_r	t_s	t_r	t_s	t_r	t_s	t_r	t_s	t_r	t_s
	sec	sec	sec	sec	sec	sec	sec	sec	sec	sec
30	72	129	77	139	82	149	88	158	93	168
40	95	171	102	184	110	197	117	210	124	223
50	119	213	128	230	137	246	146	263	155	279
60	143	256	154	276	165	295	175	315	186	334
70	166	298	179	321	192	344	205	367	218	390
80	190	341	205	367	220	393	234	420	249	446
90	214	383	230	413	247	442	264	471	280	501
100	238	426	256	458	274	491	293	524	311	556
110	261	468	282	504	302	540	322	575	342	611
120	285	510	308	550	330	589	351	628	373	667
130	310	552	333	595	357	637	380	680	404	722

Table 4.1 IMC based PID simulation results

The relationship between filter coefficient with rise time and settling time is given as

τ_p	t_r	t_s
200	$1.465\lambda-.236$	$2.62\lambda-1.14$
225	$1.6\lambda-1.38$	$2.94\lambda+1.58$
250	$1.829\lambda-.05$	$3.268\lambda+1.776$
275	$2.048\lambda-1.4$	$3.56\lambda+2.7$

300	2.187λ+.4322	3.911λ+2.639
325	2.33λ+1.67	4.25λ+.96
350	2.54λ+.31	4.56λ+1.76
375	2.74λ-.15	4.89λ+1.727
400	2.94λ-.58	5.24λ+.72
425	3.13λ-1.26	5.57λ+.56
450	3.28λ+.5	5.89λ+1.15

Table 4.2 Relation between filter coefficient and performance indices

Now for constant values of Rise Time and Settling Time, the relation between process time constant and filter coefficient is given in table below.

t_r	Relation λ=	t_s	Relation λ=
50	$70-.23\tau_p+.00024\tau_p^2$	100	$117.85-.64\tau_p+.0015\tau_p^2$
100	$139.9-.45\tau_p+.00047\tau_p^2$	200	$219.4-1.14\tau_p+.0025\tau_p^2$
150	$209.64-.68\tau_p\_.0007\tau_p^2$	300	$320-1.62\tau_p-.0035\tau_p^2$
200	$279-.9\tau_p+.00095\tau_p^2$	400	$415.6-2.06\tau_p+.0054\tau_p^2$
250	$349.3-1.1\tau_p+.0012\tau_p^2$	500	$615.6-3\tau_p+.0064\tau_p^2$
300	$562-2.8\tau_p+.006\tau_p^2$	600	$716.3-3.5\tau_p+.0074\tau_p^2$
350	$650-3.2\tau_p+.0068\tau_p^2$	700	$812.3-3.94\tau_p+.0083\tau_p^2$
400	$744-.366\tau_p+.0078\tau_p^2$	800	$916.3-4.45\tau_p+.0094\tau_p^2$

Table 4.3 Relation between filter coefficient and process time constant

Considering standard equation for rise rime to be

$$\lambda = a\tau_p{}^2 + b\tau_p + c \qquad 4.19$$

The equation of sensitivities a, b and c in terms of rise time is given by equation 4.20, 4.21 and 4.22 respectively.

$$a = 3 * 10^{-7} t_r{}^2 - 5.38 * 10^{-7} t_r + .0026 \quad 4.20$$

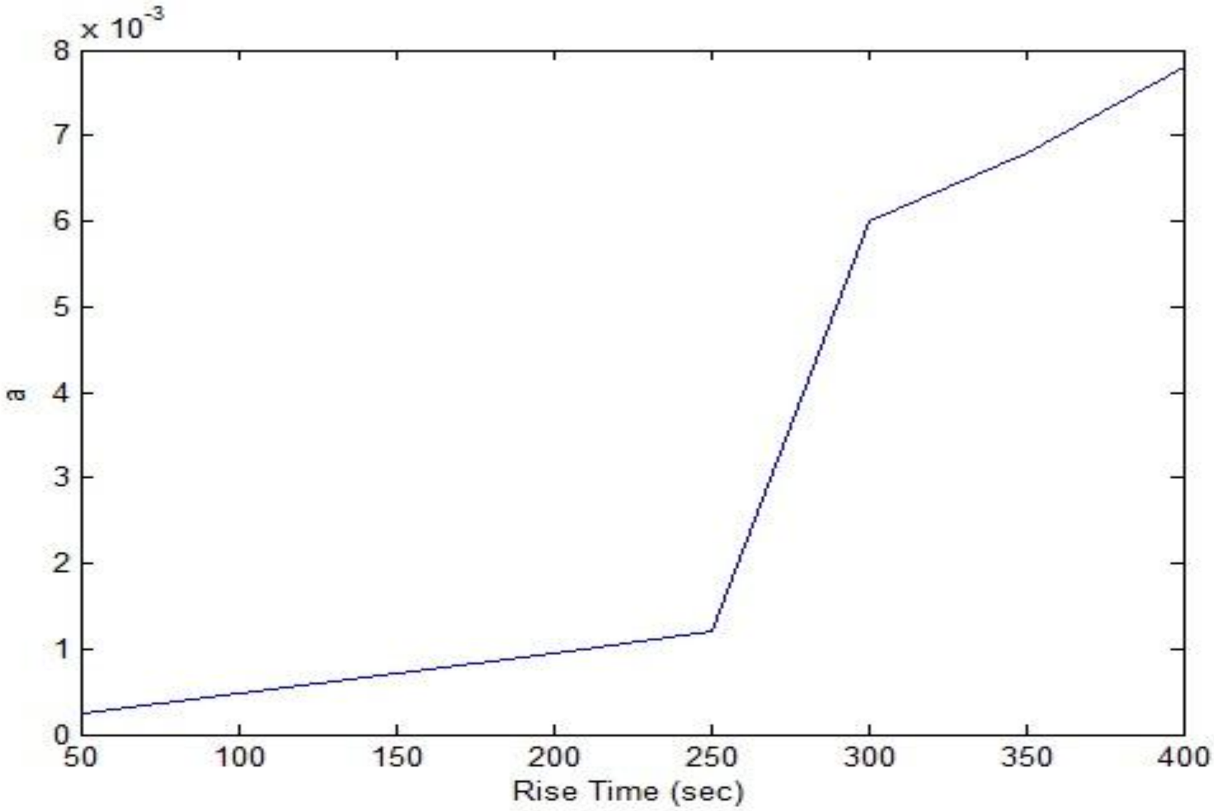

Figure 4.2 Rise Time vs a

Although the value of a is not much significant but, as we can see from figure 4.2, the variation in second order sensitivity is non-linear in shape for the rise time. The same can be seen for first order sensitivity and bias sensitivity variation with respect to Rise Time in the figure 4.3 and figure 4.4 respectively.

$$b = -9.56 * 10^{-5} t_r{}^2 + .014 t_r - .829 \quad 4.21$$

$$c = -1.39 * 10^{-5} t_r{}^3 + .0095 t_r{}^2 - .44 t_r + 81.76 \quad 4.22$$

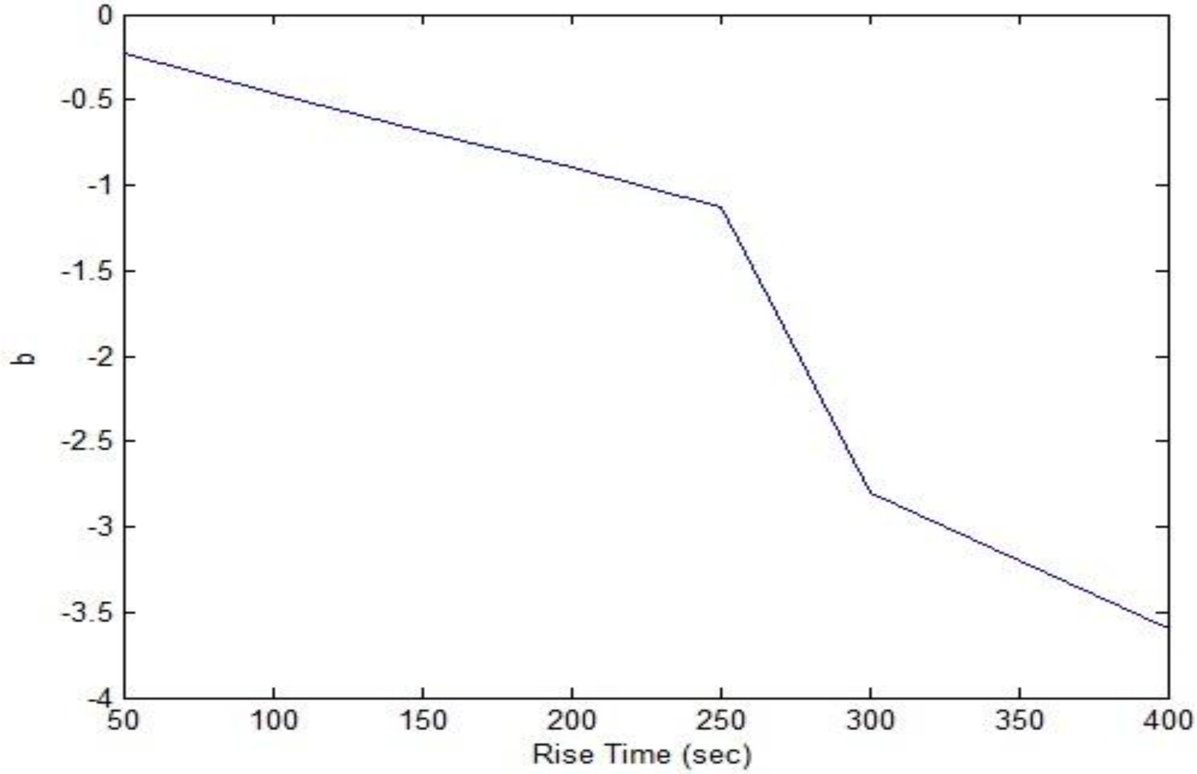

Figure 4.3 Rise Time vs b

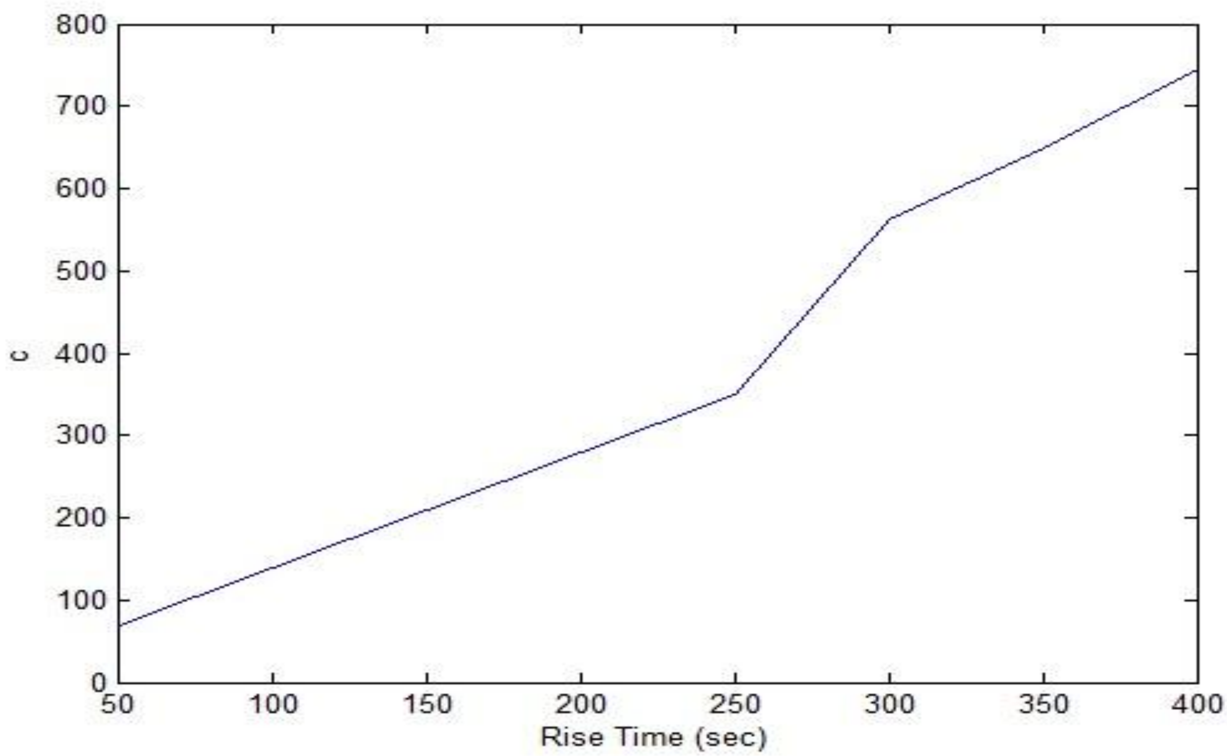

Figure 4.4 Rise Time vs c

Considering standard equation for rise rime to be equation 4.19. The equation of sensitivities a, b and c in terms of rise time is given by equation 4.23, 4.22 and 4.23 respectively.

$$a = -1.42 * 10^{-9} {t_s}^2 + 1.02 * 10^{-5} t_s + 4.9 * 10^{-4} \qquad 4.23$$

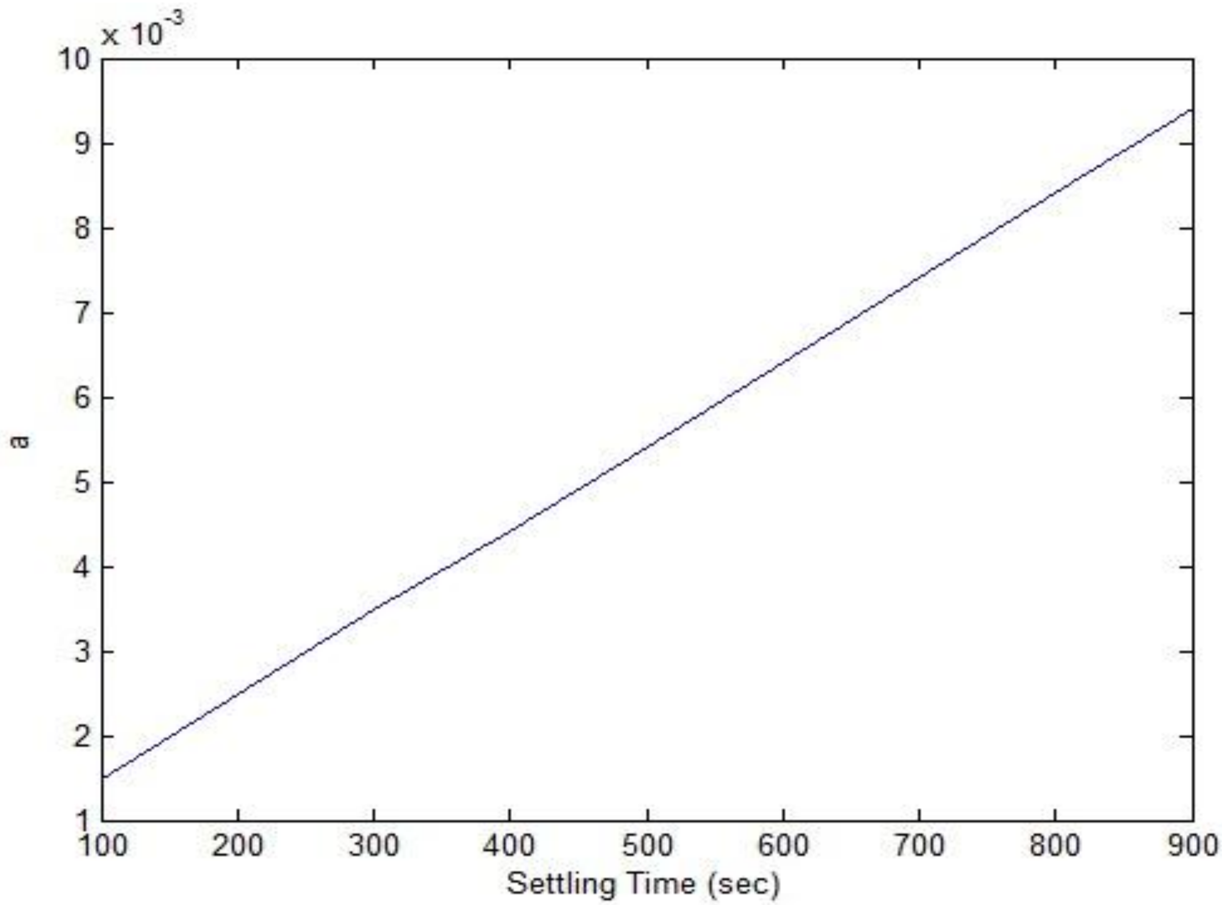

Figure 4.5 Settling Time vs a

The variation in second order sensitivity is linear for settling time as it was non-linear for rise time. The same can be seen for first order sensitivity and bias sensitivity variation with respect to Settling Time in the figure 4.6 and figure 4.7 respectively.

$$b = -1.05 * 10^{-6} {t_s}^2 - 0.005 t_s - .138 \qquad 4.24$$

$$c = -9.38 * 10^{-5} {t_s}^2 + 1.03 t_s + 15.78 \qquad 4.25$$

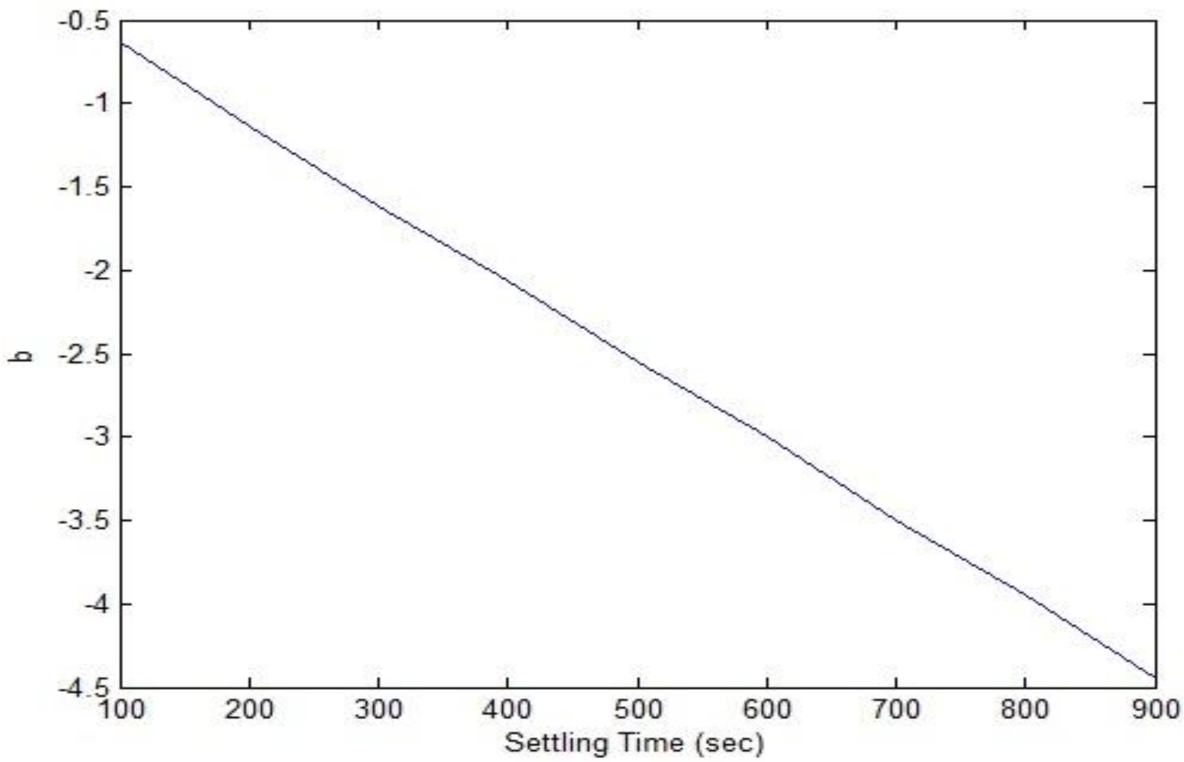

Figure 4.6 Settling Time vs b

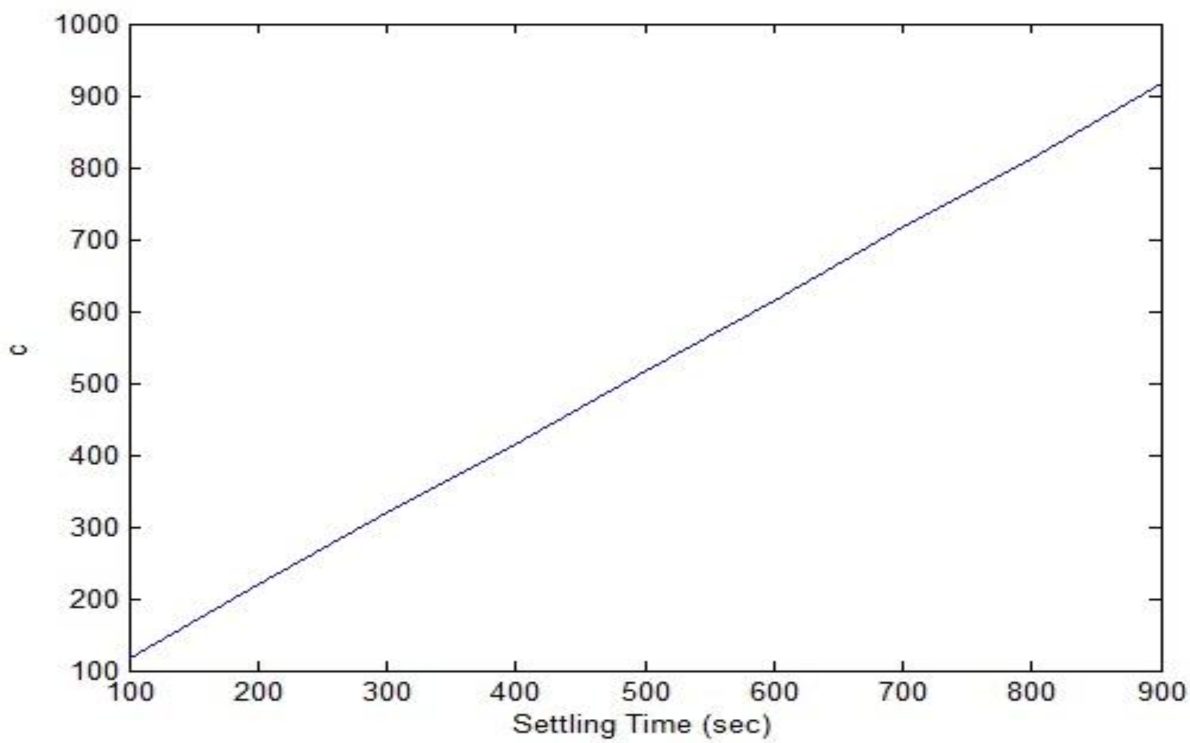

Figure 4.7 Settling Time vs c

CHAPTER-5

CONCLUSION

5.1 CONCLUSION

5.2 FUTURE SCOPE

5.1 CONCLUSION

In this project I designed IMC, IMCC, and IMC based PID controllers for multi-tank level control system with different configurations. The designing of the control loops is done using the SIMULINK software. I also proposed a tuning method for optimal tuning parameter. The method for IMC gave the optimal tuning value to be λ=70 and for IMCC to be λ_1=70, λ_2=25. The comparison between PID, PID Cascade, IMC and IMCC showed that IMCC performs much better than the other three in presence of dominant secondary disturbance. At last an empirical formula for the IMC based PID design strategy has been proposed to give the value of tuning parameter according to desires performance indices and process properties. The formula was then tested for many random transfer functions and gave a very less error.

5.2 FUTURE SCOPE

IMCC was designed for single process but by using multiple controllers, it can be extended to multi cascade system. The same controller could also be designed using any of the Artificial Intelligence method to make it more efficient and robust.

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